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Acoustics as a branch of fluid mechanics

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This article gives a review of six areas of current activity and importance in aeroacoustics, including (i) the generation of sound and vorticity by vorticity and sound, respectively, (ii) the basis for, and consequences of, the application of a Kutta condition in unsteady leading- and trailing-edge flows, and (iii) the suppression or amplification of broadband hydrodynamic and acoustic fields in a jet under the influence of weak discrete tone forcing. The intention is also to promote acceptance once again of acoustics as a serious branch of fluid mechanics.

CONTENTS

1. Introduction

In the hundred years since the appearance of the first edition of Lord Rayleigh's celebrated *Theory of Sound* (1877), the science of acoustics has departed from its original place as a branch of fluid mechanics, developed into many areas quite unrelated to fluid mechanics, and then returned again (in part, at any rate) to that field as a modern, vigorous branch. Acoustics was in Rayleigh's time the science dealing, in the main, with small disturbances with characteristic frequencies in the audible range, to air and water of substantially uniform properties. It was generally assumed that the primary means of acoustical excitation would be mechanical, and hence the extensive treatment of structural vibration in Rayleigh's treatise. There is, however, a clear antecedent of modern aeroacoustic theory in Art. 296, where a forced 'Lighthill wave equation' is derived to predict the scattered radiation from a compact density inhomogeneity. Further antecedents of modern acoustic ideas are to be found in the sections on jet and shear-layer stability, while there are also hints of the many directions subsequently taken by acoustics which are much less intimately related to the mainstream of fluid mechanics.

By the turn of the century, it appeared that little remained to be done in acoustics, as far as ordinary media under ordinary conditions were concerned. A few major ideas emerged over the next fifty years in what might be called the fluid-mechanical aspects of acoustics, notably Taylor's (1910) theory of the structure of a weak shock wave and the remarkable paper by Fay (1931) on stable periodic nonlinear waves in a thermoviscous gas, in which, as in Taylor's solution, thin weak shocks separate essentially inviscid flows and have a structure in which a balance is struck between convective nonlinearity and diffusion. The effects under discussion here are fundamental in fluid mechanics (although Fay's paper is hardly known outside the nonlinear acoustics community), and a few more such examples were thrown up by acoustics prior to 1950. To be sure, acoustics itself developed at a rapid pace throughout that time, but generally by diversifying into areas increasingly remote from those of Rayleigh's time. The topics of speech communication, phonetics, physiological acoustics, psychoacoustics, ultrasonics, physical acoustics (largely the inference of the bulk properties of *solid* matter from study of the propagation in the solid of shear and compression waves), noise and its objective and subjective quantification, signal processing... came to the forefront in acoustics, and still form the subjects of intensive research. These have tended to take acoustics into physics in parallel with fluid mechanics, rather than as a branch of it. Mathematical problems in the theory of acoustic diffraction of course remained throughout this time, as they do today, but otherwise the pace of research in conventional acoustics had become slow indeed by 1950.

That situation has been transformed in the last thirty years with the development of the subjects of aero- and hydro-acoustics. Acoustics has, naturally, continued to expand along many lines, but it now has again a branch which relies heavily on serious ideas in fluid mechanics, and, moreover, continues to throw up research topics in fundamental fluid mechanics. The originator of aero- and hydro-acoustics was one of the founding associate editors of this journal, Sir James Lighthill, and the *Journal* itself must take some credit for the partial rehabilitation of acoustics as a branch of fluid mechanics through the publication of some 150 papers in this area, the great majority of these in the last few years.

The aim of this article is to present six instances of the interaction between aeroacoustics and fluid mechanics in some of the most topical areas. All the examples discussed concern, in one way or another, the interaction between vorticity, sound and solid sharp-edged surfaces in high-Reynolds-number unsteady flows, topics far removed from classical structural acoustics. It would have been quite possible to have discussed a number of other topics which would, individually, equally well illustrate the strong position of modern acoustics in fluid mechanics; examples which I considered were nonlinear waves in bubbly liquids, the instability of free shear layers (implicitly related to most of the topics in this article), the acoustic field known as 'shock-associated noise' arising from the interaction between shear-layer turbulence and the cellular steady wave pattern in the exhaust of an underexpanded supersonic jet, the response of airfoils and turbomachinery cascades to incident vortical, entropic and acoustic disturbances, and others. I felt, however, that it was important that the various topics have some fairly close relationship with each other, beyond illustrating the interplay between acoustics and fluid mechanics. The six examples selected do represent both this interplay and the current state of aeroacoustics rather well. There is, of course, no attempt here to provide anything more than an ephemeral (and incompletely documented) view of our understanding of these fascinating fluid-mechanical phenomena.

2. Generation of sound by vorticity

The aim of aeroacoustic theory is firstly to provide a reliable means of predicting as many features as possible of the sound generated by turbulent flow from the crudest specification of that flow (the original aim), and secondly (and more recently), to provide a framework for the solution of model problems involving the interaction between simple vortical and compressible motions in order that particular mechanisms can be explored in detail. Lighthill (1952) was concerned with the former, and he set the pattern for most subsequent theories in formulating an analogy between the actual situation (e.g. the generation of broadband noise by a high-speed jet exhaust) and a simple classical acoustic situation on which the turbulent flow acts as an externally imposed source distribution with a particular structure. Such analogies have also been used extensively to solve model problems, because the direct calculation of unsteady, vortical, compressible flows can only be accomplished in a very few cases (perhaps the most notable to date involving the Kelvin vortex of finite cross-section, neutrally stable in strictly incompressible flow, unstable, despite radiation damping, in weakly compressible flow; see Broadbent & Moore 1979).

Lighthill's theory has been by far the most successful and versatile. It has served as the basis for innumerable schemes for the prediction of mixing noise from jet aero-engines and of boundary-layer and wake noise from underwater vehicles, and at the other extreme has often featured in theories of the heating (by dissipation of the energy of aerodynamically generated acoustic waves) of the solar corona. Although devised in the first instance for subsonic flows, it has been extended to cover the entire Mach number range of engineering interest, and has been generalized to include the effects of solid boundaries, either at rest or in arbitrary motion, those of convective amplification and fluid shielding by the mean flow, and the phenomena associated with inhomogeneities in the turbulence – such as temperature inhomogeneities, bubbles in liquids and dust particles in gases. For reviews of these, and many other aspects, the reader is referred to Ffowcs Williams (1969, 1977), Crighton (1975) and Goldstein (1976). Lighthill's theory has also been used to calculate the radiation from certain unsteady flows for which an exact solution is known in the incompressible limit, but for these purposes it is neither the most efficient nor the most illuminating procedure.

Consider an unsteady flow confined to a bounded region; we wish to predict the sound field radiated to large distances outside the flow. An acoustic analogy for this problem consists of a forced wave equation

$$\left(\frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)h = q \tag{2.1}$$

(where a_0 is the sound speed in the quiescent fluid, h a field variable which in the distant field can be identified with the pressure perturbation) together with a specification of the source q in terms of quantities which can be estimated independently of the sound field from a knowledge of the unsteady flow. All choices for h and q consistent with the equations of mass, momentum and energy conservation must lead to the same sound field if all the flow variables are known exactly; the advantage of one choice over another is then entirely dependent on the details of the particular problem. For example, if we have only a crude specification (in terms of intensities and length scales, say) of a turbulent flow, Lighthill's choice should be followed, as this is designed to eliminate the gross errors that can easily arise from incomplete knowledge of the flow. If, on the other hand, we consider low-Mach-number flows with concentrated vorticity known in analytical form, then an appropriate choice would involve a q related as closely as possible to the vorticity alone.

Infinitely many choices of acoustic analogy are, of course, possible. Three simple analogies of type (2.1) have proved popular. They are due to Lighthill (1952), Powell (1964) (greatly extended by Howe (1975)) and Ribner (1962), and have, respectively,

$$q = \partial^2 T_{ij}/\partial x_i \partial x_j, \tag{2.2}$$

$$q = \rho_0 \operatorname{div} \mathbf{L} \tag{2.3}$$

and

$$q = -\frac{1}{a_0^2}\frac{\partial^2 p^{(0)}}{\partial t^2}, \tag{2.4}$$

in which

$$T_{ij} = \rho u_i u_j + p_{ij} - a_0^2 \rho \delta_{ij} \tag{2.5}$$

defines Lighthill's quadrupole stress tensor in terms of velocity $\mathbf{u}$, stress p_{ij} and density ρ,

$$\mathbf{L} = \boldsymbol{\omega} \wedge \mathbf{u} - T \operatorname{grad} S \tag{2.6}$$

is the Lamb vector of the Powell–Howe theory, with $\boldsymbol{\omega}$ the vorticity, T the temperature and S the entropy, and

$$\nabla^2 p^{(0)} = -\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{2.7}$$

defines Ribner's pseudo-sound pressure $p^{(0)}$. The number of space derivatives present in q determines the integral-vanishing properties of the source, and hence its multipole structure; thus (2.2)–(2.4) represent, respectively, quadrupole, dipole and monopole source distributions. A point multipole (or one in which the constituents are separated by a scale l small compared with the radiated wavelength λ) has a radiation efficiency and directivity pattern quite different from those of the individual simple sources which comprise it, and therefore the three approaches can be reconciled only when the integrated effect of the whole distribution is calculated from the Green's function solution

$$h(\mathbf{x}, t) = \int G(\mathbf{x}, t; \mathbf{y}, \tau)\, q(\mathbf{y}, \tau)\, d^3\mathbf{y}\, d\tau \tag{2.8}$$

to (2.1). If the distribution is known in sufficient detail that reconciliation is merely a matter of integration and algebra; if it is not, the use of an approximate $p^{(0)}$ in an acoustically efficient monopole representation like (2.4) will lead to a much larger estimate (by a factor $O(\lambda/l)$) for h than if (2.3) were approximated, and that in turn to a comparable overestimate relative to that resulting from approximation to Lighthill's quadrupole T_{ij}. This sensitivity of some representations to fine detail of the source is heightened by the very different behaviour of the q's far from the flow region; T_{ij} decreases like $|\mathbf{x}|^{-6}$ if incompressible values are used, $\mathbf{L}$ vanishes identically outside the region containing vorticity, and $p^{(0)}$ decreases like $|\mathbf{x}|^{-3}$. Thus the total instantaneous source strength $\int q(\mathbf{y}, \tau)\, d^3\mathbf{y}$ vanishes for (2.2) and (2.3), but is infinite for (2.4)!

Now the ability to estimate instantaneous source integrals is important, for in the retarded-potential solution to (2.1)

$$h(\mathbf{x}, t) = \frac{1}{4\pi}\int q\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_0}\right)\frac{d^3\mathbf{y}}{|\mathbf{x}-\mathbf{y}|} \tag{2.9}$$

one does not usually know enough about the *variations* in q with retarded-time $t - |\mathbf{x}-\mathbf{y}|/a_0$ (when $\mathbf{y}$ varies over a typical coherent turbulence source, or 'eddy') to predict the correct value between zero and infinity. If we think of the eddy as having a length scale l and velocity scale u, then the maximum retarded-time difference is of order l/a_0, a time far shorter (at low Mach numbers $m = u/a_0$) than the time-scale l/u over which changes can usually be assessed in turbulent flow. We therefore need a formulation which is insensitive to retarded-time differences – which (2.4) is evidently not, as was forcefully pointed out by Lighthill (1963) and Crow (1970).

Lighthill's form, with its quadrupole structure, takes explicitly into account the twofold integral-vanishing property that all aeroacoustic sources must have, because of mass and momentum conservation in regions free of boundaries, and leads to a far-field form

$$h(\mathbf{x}, t) \sim \frac{x_i x_j}{4\pi a_0^2 |\mathbf{x}|^3}\frac{\partial^2}{\partial t^2}\int T_{ij}\left(\mathbf{y}, t - \frac{|\mathbf{x}-\mathbf{y}|}{a_0}\right) d^3\mathbf{y} \tag{2.10}$$

in which retarded-time variations can be safely neglected for $m \ll 1$, yielding finite, and usually non-zero, estimates in the form of valuable scaling laws based on crude dimensional representations of T_{ij}. This completely accounts for the fact that such a simple acoustic analogy has proved so robust, and capable of revealing so many features of the sound field; see, for example, the demonstration by Ffowcs Williams (1969) that Lighthill's q transforms naturally into a dipole or even monopole form when the flow contains inhomogeneities in composition or heat content.

Dimensional analysis of (2.10) leads readily to the estimate

$$(\rho_0 u^3 l^2)\left(\frac{V}{l^3}\right) m^5 \tag{2.11}$$

for the acoustic power output of uncorrelated eddies, each of volume l^3, distributed throughout a volume V, in which the first term $(\rho_0 u^2)(l^3)(u/l)$ represents the rate of supply of energy to an eddy of scale l to maintain its motion at speed u, the second represents the number of eddies in V and the third expresses the quadrupole inefficiency of aeroacoustic sources at low Mach number. Equation (2.11) is equivalent to the celebrated 'U^8-law' for the increase in acoustic power with jet exhaust speed U. The many further results of this theory do not concern us here, nor any of the attempts to 'improve' upon (2.11) in the sense of finding the numerical coefficient involved. In the first place there is no reason to expect such a coefficient to be universal, because sound generation involves the low- rather than high-wavenumber components of turbulence, and hence there can be no such thing as a formula for 'the acoustic power output of unit volume of turbulence'. And in the second, even for a given turbulent shear flow, the value of the constant rests upon details of the flow which have not yet been measured, nor are ever likely to be; specifically, on the integral, first over spatial separation and then over the turbulent volume, of the fourth time derivative of the space–time covariance of the Reynolds stresses (81 terms), an experimental task of great difficulty and effort, and of almost no value whatever. Jet noise prediction schemes commonly postulate a functional form for the said covariance, with a few disposable parameters taken from shear flow measurements or adjusted to fit the noise prediction to measured data, but, principally because of the fourth (i.e. quadrupole) derivative, the numerical coefficient is indeed sensitive to the functional form assumed. Despite all this, however, the scaling laws like (2.11) are already of considerable value as they stand, particularly in underwater acoustics. There m never exceeds 10^{-3}, so that an incorrect source modelling leading to, say, a dipole source, would lead to an overestimate of the intensity by a factor $m^{-2} \sim 10^6$ – which is more likely to be a serious error than any arising from inadequate knowledge of the numerical coefficients involved. Further, although derived on the basis of low-Mach-number asymptotics, the scaling law (2.11) holds, in the case of jet noise from carefully controlled model rigs, over the wide range $0{\cdot}3 \lesssim U \lesssim 2a_0$ of exhaust velocities U.

Two different 'analogies' have been proposed to deal with high Mach numbers. These are due to Phillips (1960) and Lilley (1972) (see Goldstein 1976). Phillips' approach involves a *convected* wave equation

$$\left\{\frac{D^2}{Dt^2}-\frac{\partial}{\partial x_i}\left(a^2\frac{\partial}{\partial x_i}\right)\right\}\ln(p/p_0) = \gamma\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}, \tag{2.12}$$

which has the merit of extracting density from the Lighthill source (arguably the most significant manifestation of compressibility) and leaving a source specified solely in terms of velocities. Lilley objected to (2.12) on the grounds that as fluctuation amplitudes become small all source terms on the right should vanish at least quadratically so that the equation should then describe freely propagating disturbances to the mean shear flow. That property (in no way mandatory) is not possessed by Phillips' equation, which contains a linear 'shear refraction' term on the right. Lilley re-wrote Phillips' equation in the form

$$\frac{\bar{D}}{Dt}\left\{\frac{\bar{D}^2}{Dt^2}-\frac{\partial}{\partial x_i}\left(\bar{a}^2\frac{\partial}{\partial x_i}\right)\right\}\ln(p/p_0)+2\frac{\partial \bar{u}_1}{\partial x_3}\frac{\partial}{\partial x_1}\left(\bar{a}^2\frac{\partial}{\partial x_3}\right)\ln(p/p_0)$$
$$= \text{terms at least quadratic in fluctuations.} \tag{2.13}$$

The left-hand side of (2.13) is, of course, just the compressible form of the Rayleigh operator determining the *instability* properties of the mean flow – which has led to considerable controversy as to the role which should be played in the theory by these instabilities. More basic than this, however, is the point that if one is interested in finding the sound field generated by turbulent perturbations to a mean flow it is simply not possible to derive Lilley's equation by any rational scaling and expansion method, because the time scales involved do not permit the use of the *mean* velocity and temperature profiles on the left of (2.13) (see Crighton 1979). A further difficulty is that equations such as (2.12) and (2.13) are not easily solved in analytical form, while if the mean profile is used in (2.13) some rather artificial sources must be included on the right to account for the fact that throughout the brief time a sound wave takes to traverse a shear layer the shear-layer profile does not remotely resemble the mean. All in all, although (2.12) and (2.13) constitute exact analogies, they are taken with respect to a reference situation (left-hand side equal to zero) of questionable relevance and, in my opinion, have yet to produce a result of widely accepted validity which has not been obtained more explicitly from generalizations of Lighthill's theory (e.g. those due to Dowling *et al.* 1978).

At low Mach numbers Lighthill proposed the approximation $T_{ij} \approx \rho_0 v_i v_j$, where $\mathbf{v}$ is the velocity field of incompressible flow. In that limit, Lighthill's theory amounts to a theory of sound generation by *vorticity*, for $\mathbf{v}$ can be defined entirely in terms of $\boldsymbol{\omega} = \operatorname{curl} \mathbf{v}$ by the Biot–Savart law

$$\mathbf{v}(\mathbf{x}, t) = \frac{1}{4\pi} \int \frac{\boldsymbol{\omega}(\mathbf{y}, t) \wedge (\mathbf{x}-\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^3} d^3\mathbf{y}. \tag{2.14}$$

However, when one thinks of the low-Mach-number situation as comprising an inner vortical core of scale l and an outer compressible acoustic mantle of scale $\lambda \sim lm^{-1}$, the formal connection established between h and $\boldsymbol{\omega}$ seems rather remote. A closer connection was provided by Powell, who wrote

$$\frac{\partial^2 \rho_0 v_i v_j}{\partial x_i \, \partial x_j} = -\frac{\partial}{\partial x_i}\{\rho_0 (\mathbf{v} \wedge \boldsymbol{\omega})_i\} + \nabla^2(\tfrac{1}{2}\rho_0 \mathbf{v}^2) \tag{2.15}$$

and argued that (as can be confirmed from Crow's (1970) analysis) the second term on the right gives rise to a much smaller sound field than the first – and hence equation (2.3), if entropy gradients can be neglected. The Lamb vector $\rho_0(\boldsymbol{\omega} \wedge \mathbf{v})$ can be regarded as the Kutta–Joukowski force on a material line element; momentum conservation demands that these lift forces be arranged in cancelling pairs, so that Powell's vortex dipole actually has a hidden quadrupole structure which must be explicitly brought out in applications to ill-defined sources in order to avoid a serious overestimate of the sound. This makes it less convenient than Lighthill's form for most general purposes, though it is much *more* convenient for the solution of model problems, where most exact solutions for the incompressible field $\mathbf{v}$ involve vorticity concentrations in lines, planes or rings. Then one can utilize the delta-function behaviour of $\boldsymbol{\omega}$ effectively, without needing approximations to the source function – except that the value of $\mathbf{v}$ to be used in $(\mathbf{v} \wedge \boldsymbol{\omega})$ is taken as the velocity of the vortex filament, etc., ignoring the infinite contribution to $\mathbf{v}$ which is self-induced by the local vorticity.

A significant generalization of Powell's approach was achieved by Howe (1975) who observed that an analogy could be framed in which the quiescent acoustic medium of the analogies represented by (2.1) could be replaced by a steady irrotational mean flow. The propagation operator for irrotational perturbations to such a flow is

$$\left\{\frac{D}{Dt}\left(\frac{1}{a^2}\frac{D}{Dt}\right) + \frac{1}{a^2}\frac{D\mathbf{u}}{Dt}\,.\nabla - \nabla^2\right\} \equiv \mathscr{L} \tag{2.16}$$

say, where a is the local sound speed, D/Dt the derivative following the total velocity $\mathbf{u}$. In the presence of vorticity, and entropy gradients, there is no potential on which $\mathscr{L}$ can act; Howe showed that the stagnation enthalpy

$$B = \tfrac{1}{2}\mathbf{u}^2 + \int \frac{dp}{\rho} + \int T\,dS \tag{2.17}$$

is the natural variable to use in place of $\partial\phi/\partial t$ (and corresponding to h of (2.1)) and that then

$$\mathscr{L}B = \operatorname{div}\{\boldsymbol{\omega} \wedge \mathbf{u} - T \operatorname{grad} S\} - \frac{1}{a^2}\frac{D\mathbf{u}}{Dt}\,.\{\boldsymbol{\omega} \wedge \mathbf{u} - T \operatorname{grad} S\}. \tag{2.18}$$

This equation contains Powell's result in the low-Mach-number, uniform entropy limit, includes thermodynamic inhomogeneity sources in the Lamb dipole, and includes all effects associated with a mean flow as long as it is a *potential* mean flow (and not the transversely sheared flows attacked by the Lilley formulation). As such it is more suited to internal duct and engine flows than external jet flows, but it has been applied by Howe himself to a great variety of problems involving sound generation by vortical and entropic concentrations propagating in inhomogeneous mean flows. As examples we quote his (1975) nonlinear theory of vortex shedding and sound production in flute-like instruments, the generation of pressure waves by the interaction of vorticity waves propagating on swirling duct flow with a contraction or nozzle (Howe & Liu 1977) and the theory of shock-associated noise produced when shear-layer vorticity interacts with the steady cellular pattern in a supersonic jet exhaust (Howe & Ffowcs Williams 1978).

The Powell–Howe theory is the nearest we have come to relating sound sources to the vorticity – where by 'sources' we mean the q on the right side of a differential acoustic analogy like (2.1). However, the sources are not locally related to the vorticity, and in fact are *non-locally, nonlinearly* related to the vorticity through (2.3), (2.6) and (2.14). The only means yet found of relating the sound field linearly to the vorticity is due to Möhring (1978), who not only transformed the expression for the source q using the nonlinear Helmholtz vortex laws, but transformed the Green's function with which the source is convolved. In the Green's function solution

$$h(\mathbf{x},t) = \int \rho_0 G(\mathbf{x},t;\mathbf{y},\tau)\frac{\partial L_i}{\partial y_i}(\mathbf{y},\tau)\,d^3\mathbf{y}\,d\tau$$

to the Powell–Howe wave equation we integrate by parts, giving

$$h(\mathbf{x},t) = -\rho_0\int \frac{\partial G}{\partial y_i}(\mathbf{x},t;\mathbf{y},\tau)\,L_i(\mathbf{y},\tau)\,d^3\mathbf{y}\,d\tau,$$

define a vector Green's function by

$$\frac{\partial G}{\partial y_i} = (\operatorname{curl}\mathbf{G})_i$$

and again integrate by parts to give

$$\begin{aligned} h(\mathbf{x},t) &= \rho_0\int G_i(\operatorname{curl}_y\mathbf{L})_i\,d^3\mathbf{y}\,d\tau \\ &= -\rho_0\int G_i(\mathbf{x},t;\mathbf{y},\tau)\frac{\partial\omega_i}{\partial\tau}(\mathbf{y},\tau)\,d^3\mathbf{y}\,d\tau \\ &= -\rho_0\frac{\partial}{\partial t}\int G_i(\mathbf{x},t;\mathbf{y},\tau)\,\omega_i(\mathbf{y},\tau)\,d^3\mathbf{y}\,d\tau \qquad (2.19)\end{aligned}$$

after a further integration by parts and use of the dependence of both G and $\mathbf{G}$ on $(t-\tau)$ only. Note that, whereas $\mathbf{L}$ involves both vorticity and entropy gradient terms, (2.19) expresses the field *entirely* in terms of $\boldsymbol{\omega}$, use having been made of Helmholtz' equation in the form $\partial\boldsymbol{\omega}/\partial t + \operatorname{curl}\mathbf{L} = 0$.

The price paid for the attractive relation (2.19) is twofold. First, since G_i is not the usual retarded-potential scalar Green's function, (2.19) does not have the properties we intuitively expect in a source–sound relation. Second, as Möhring remarks, the principal difficulty with (2.19) is that in general the vector $\mathbf{G}$ does not exist, for the integrability condition for the set $\operatorname{curl} \mathbf{G} = \operatorname{grad} G$ is $\nabla^2 G = 0$, which is not usually satisfied. Indeed, when G is symmetric in its arguments it satisfies

$$\nabla_y^2 G = \frac{1}{a_0^2}\frac{\partial^2 G}{\partial \tau^2} - \delta(\mathbf{x}-\mathbf{y})\,\delta(t-\tau);$$

but Möhring notes that the delta functions vanish in the usual case where $\mathbf{x}$ is in the far field and $\mathbf{y}$ in the source region, while the first term on the right is presumably small at low Mach numbers – so that then the requisite $\mathbf{G}$ might almost exist! Möhring actually gives several examples in which $\mathbf{G}$ can be found; in one G is harmonic in $\mathbf{y}$ so that there is no difficulty (and this always happens if $\mathbf{y}$ lies within a wavelength or so of a scattering body), while in another $\nabla_y^2 G \neq 0$ but it is *still possible* nonetheless to find a $\mathbf{G}$ with the right integral properties in conjunction with the Lamb vector $\mathbf{L}$. He also gives a succinct representation of the radiated field of any number of compact vorticity blobs, involving quantities like the vortex impulse and vortex energy familiar in vortex dynamics, and an expression for the intensity of turbulence-generated noise which is linear in the vorticity covariance, and hence linear in the velocity covariance, in contrast to the Lighthill expression which involves the mean product of four velocities (and hence the square of the velocity covariance if a quasi-Gaussian hypothesis is invoked).

Because $\mathbf{G}$ always exists when G is harmonic in $\mathbf{y}$ it is natural (though perhaps only as a formal exercise) to recast Möhring's formulation in terms of matched expansions (for $m = u/a_0 \to 0$). That is, Möhring's method is used for incompressible flow to relate the pressure fluctuation linearly to the vorticity of a compact flow, and this fluctuation is then matched to an outer compressible wave field, an approach recently used in many applications by Obermeier (1979). Matching procedures of this kind (and, quite generally, for low-frequency acoustic problems) are not altogether straightforward, however, as shown by the penetrating analysis of Crow (1970), and the use of (2.19) directly is preferable whenever possible.

Many further applications will doubtless be found for the inhomogeneous wave equations mentioned here, and also for the very different representation (2.19). Each in its way represents a theory of sound generation by vorticity; the Lighthill theory is quite the most successful in leading to general results, the Powell–Howe version in enabling model problems to be solved. Neither involves the vorticity as simply and explicitly as one might hope, but the prospect of finding a preferable differential analogy seems slim. If one enlarges the idea of an analogy to cover expressions like (2.19) then the search for an analogy couched in terms of vorticity alone is over; but we have yet to discover (through study of the purely acoustic problem defining $\mathbf{G}$) the physical situation to which that analogy refers!

3. Generation of vorticity by sound

Sound may be generated by vorticity either in free space through the action of the nonlinear terms in the Navier–Stokes equations or through the linear coupling which takes place when these fields are together required to satisfy boundary conditions demanded by, say, the presence of an inhomogeneous surface. By the same token, vorticity can be generated by sound in either of these situations. In free space the process involved is that of stretching of fine-scale vortex lines by the weak large-scale sound field. This process – the stretching of vorticity by its own sound field – has not yet been studied, and no claim has been entered for its importance in terrestrial applications, though it could conceivably be significant in astrophysical situations, where the levels of aerodynamically generated sound might be large enough to produce a significant back-reaction on the flow responsible for them.

The second possibility, where vorticity is produced by the interaction of sound with a solid surface, leading to a significant attenuation of the sound if there is a mean flow to convect the vorticity away, has been the subject of much recent work, although Bechert (1980) gives examples of practical noise suppression devices which (perhaps unwittingly) rely on this mechanism, going back as far as 1916. The effect was first noticed in aeroacoustics in studies of high-Reynolds-number jet reponse to low-amplitude acoustic waves propagating down the jet tailpipe, and it is in that context that the most spectacular results are found. It was originally thought (Crow 1972; Crighton 1975) that the far-field sound at the frequency of an incident forcing tone could be significantly amplified by the acoustic field associated with instability waves triggered on the jet by the forcing, provided the forcing frequency lay within the instability band. This now appears to be the case *only* when the jet velocity is high enough that the instability phase speed is supersonic relative to the ambient medium, in which case relatively intense 'Mach wave radiation' is generated by the instability as it amplifies and then decays downstream (Tam & Morris 1980). Moore (1977*a*) showed experimentally that, at subsonic mean flow Mach numbers, no significant amplification of the far-field sound existed (despite very large amplifications, at the forcing frequency, of the fluctuations within the jet flow and the near field). This was done by measuring the power incident upon the nozzle exit from within the tailpipe, that reflected back down the tailpipe, and the far-field radiated power; the difference between the first two of these – i.e. the net power transmitted along the tailpipe – is denoted by W_T, the third by W_R. For a range of subsonic Mach numbers, and for Helmholtz numbers $k_0 R$ (k_0 = acoustic wavenumber at frequency ω, R = tailpipe exit radius) greater than about 1·5, the ratio W_R/W_T was found to be close to unity, whereas at lower Helmholtz numbers there was a significant imbalance, most of the incident power appearing neither in the far field, nor in the reflected waves.

A definitive set of measurements was then compiled by Bechert *et al.* (1977), who showed that the attenuation $-\Delta$, where

$$\Delta = 10 \log_{10} (W_R/W_T),$$

could be extremely large, as much as 20 dB at $k_0 R = 0{\cdot}1$ and a Mach number of 0·3! Howe (1979*a*) next provided a detailed theoretical prediction of these effects, following Munt's (1977) earlier prediction of the far-field directivity pattern produced when a tailpipe acoustic mode is diffracted and refracted out of the exit plane of a semi-infinite circular duct carrying uniform subsonic jet flow. Munt had been extremely successful in predicting the directivity, at any rate for low to moderate frequencies and for cold jets, and Howe's calculation of the attenuation is equally successful. Simpler versions of this theory were subsequently derived by Howe (1980*a*), Cargill (1979) and Bechert (1980), each taking a low-Strouhal-number limit but with slightly different accuracy in regard to the Mach number M. Howe (1979*a*) assumes low

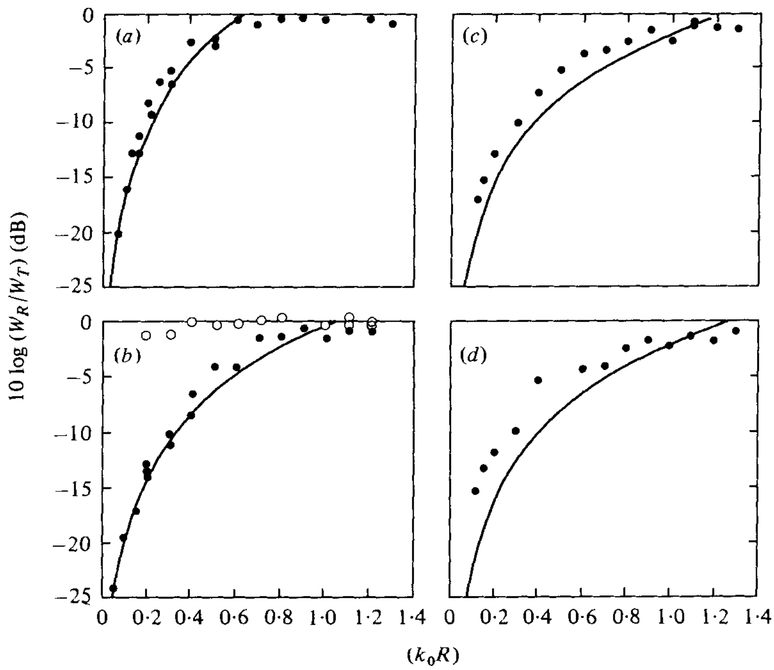

FIGURE 1. Sound attenuation as function of Helmholtz number (k_0R). Open circles in (*b*) show acoustic power conservation in absence of flow. ——, equation (3.2); ●, measured values. (*a*) $M = 0{\cdot}1$, (*b*) $M = 0{\cdot}3$, (*c*) $M = 0{\cdot}5$, (*d*) $M = 0{\cdot}7$. (From Bechert 1980 *J. Sound Vib.* **70**, p. 389, Copyright by Academic Press Inc. (London) Ltd.)

Helmholtz number, and further neglects M^2 compared with unity, but otherwise his theory relies on an exact solution of the jet/tailpipe interaction problem in which, as in Munt's work, the satisfaction of a Kutta condition on the unsteady flow at the duct lip is essential (see §4 below). A simplified version of this theory (Howe 1980*a*) ignores all terms $O(M)$ relative to unity, with the result

$$\frac{W_R}{W_T} = \frac{(k_0R)^2}{4M}. \tag{3.1}$$

Bechert's (1980) theory does not solve a boundary-value problem, but assumes instead that the sound field can be regarded as generated by monopole and dipole sources on the exit plane, the two being coupled by the postulate that in the presence of mean flow the open-end pressure reflection coefficient remains -1, as in the absence of flow. Cargill (1979) shows that postulate to be equivalent to the application of a Kutta condition. He also shows that for low Helmholtz numbers and *finite* Mach numbers the sources considered by Bechert should be supplemented by certain quadrupoles in the jet flow, so that the M^2 term in Bechert's (1980) equation (15),

$$\frac{W_R}{W_T} = \frac{(1+\frac{4}{3}M^2)}{4M}(k_0R)^2 \tag{3.2}$$

is incorrect and should be replaced by

$$\frac{W_R}{W_T} = \frac{(1+\frac{1}{3}M^2)}{4M(1-M^2)^3}(k_0R)^2. \tag{3.3}$$

In figure 1 we reproduce figure 3 of Bechert (1980) which gives a comparison of (3.2) with measurement for $M = 0{\cdot}1$, $0{\cdot}3$, $0{\cdot}5$ and $0{\cdot}7$. The agreement is good, except at the higher Mach numbers where the incorrect M^2 term in (3.2) must be significant. The effect of the change represented by Cargill's (3.3) above is to *raise* the curves in this figure by 0·1 dB, 0·9 dB, 2·8 dB and 7·2 dB, uniformly in $(k_0 R)$, for the four respective values of M. At $M = 0{\cdot}5$ and $0{\cdot}7$, and $k_0 R \lesssim 0{\cdot}6$, this leads to a much better prediction of the data than that offered by (3.1) or (3.2).

The Kutta condition, of finite velocities at the duct lip, will be discussed further in §4; it serves to fix the rate of vorticity shedding from the lip into the downstream shear layers in terms of the incident tailpipe wave amplitude, and the kinetic energy flux associated with these vortical fluctuations accounts precisely for the 'attenuation' experienced by the incident wave. In the vortex sheet jet models the vortical fluctuations take the form of spatially amplifying instability waves; however, this unbounded exponential amplification is totally irrelevant to the energy conversion process, which is a purely local one, at the lip. At the low frequencies of interest here the growth rates of the instabilities are very low, and Howe (1979*a*) has given an alternative modelling in which finite shear-layer thickness leads only to neutral convection of the vorticity with very little change in the acoustic attenuation.

In Munt's analysis (and in several papers preceding it) emphasis was placed on the need for a *causal* solution in time-harmonic problems where spatial instability is possible. When doubly infinite shear-layer problems are considered, the causality requirement seems to lead to a unique solution (one unbounded downstream); in practice, however, shear layers are shed from splitter plates or nozzles, and in such cases causality does not lead to a unique solution. Crighton & Leppington (1974) showed that, for the two-dimensional flow past an upstream splitter plate the solution satisfying a Kutta condition at the trailing edge was automatically causal, but that there were also causal solutions with singular velocity fields at the edge. The same is true of the jet–tailpipe problem; the properties of the Kutta condition solution (necessarily causal) have just been described, but there are other causal solutions, including one in which there is *no* vorticity shedding from the pipe lip. Then (Cargill 1979) the pressure reflection coefficient magnitude becomes $(1+M)/(1-M)$ at low $k_0 R$ and almost all the incident energy is then reflected back down the tailpipe, only a small fraction, $O(k_0^2 R^2)$, radiating to the far field.

The causality requirement may still turn out to be crucial if (as appears to be the case for hot jets) several complex instability wavenumbers appear to correspond to a given frequency (some of which might, under the causality requirement, turn out to be spurious and non-physical), but for the cold flow problems discussed here it appears that for trailing-edge flows the overriding condition relates to the rate of vorticity shedding at the edge or lip, as expressed in the Kutta condition.

Vortex shedding also has an interesting effect on the magnitude of the open-end pressure reflection coefficient; for low $k_0 R$ this magnitude is 1, and it then rises above 1, reaches a maximum and falls to zero as $k_0 R \to \infty$, while remaining everywhere larger than the value for zero mean flow. The maximum is reached for values of M and $k_0 R$ which give the *Strouhal number St* ($= \omega R/\pi U$) a value around 0·5 – close to the 'preferred jet column instability mode' Strouhal number (see §§6, 7). This behaviour cannot be predicted by theories like those leading to the $(k_0 R)^2$ variation of (3.1)–(3.3); either numerical evaluation of Munt's theory is needed (Munt 1981), or analysis to higher order in $k_0 R$ (Cargill 1981). The two approaches agree very closely with each other, and reproduce the experimental results reasonably well. This further confirms the general understanding of acoustic–hydrodynamic energy conversion at a jet pipe exit; in particular, approximate theories which ignore the vortex shedding and jet flow instabilities induced by the Kutta condition fail completely to describe the reflection coefficient behaviour with $k_0 R$ in the presence of flow (see references cited above).

The low-frequency attenuation may occur wherever acoustic waves and mean flows are coupled at a trailing edge, from which vorticity shedding can take place at the expense of the acoustic input. The mean flows do not have to be different on the two sides of the splitter plate or duct, nor do the flows have to be grazing flows. For example, Howe (1980*b*, *c*) has examined the reflection and transmission of sound waves by a plate perforated with either two-dimensional slits or circular apertures, with the same grazing flow on both sides, and also (1979*b*) with a bias flow normal to the plate. In each case the trailing-edge velocities are required to be finite (which makes for an interesting problem for circular apertures in a grazing flow, with the Kutta condition imposed only on the upstream semicircle) while those at leading edges are allowed to be infinite in the usual inverse-square-root fashion (though see § 5 below). Sound generation by vorticity does of course occur when the vorticity fluctuations in an aperture interact with the leading edge of the next section of plate, but this is taken into account in the calculation and the net result is always an attenuation of sound by the screen. A typical result (§ 4 of Howe 1980*b*) is the following expression for the net power loss per unit area, for waves normally incident on a screen with slit perforations, at low aperture Strouhal number in the presence of grazing flow at Mach number M:

$$\Delta = \frac{(\frac{1}{2}\pi\sigma)^2 + M^2}{(\frac{1}{2}\pi\sigma + M)^2}, \tag{3.4}$$

with σ the open-area ratio. This has a minimum value of $\frac{1}{2}$, corresponding to an attenuation of 3 dB, at $M = \frac{1}{2}\pi\sigma$. Thus the attenuation here is much less dramatic than in the jet exhaust problem (where there is no re-conversion of vortical energy into sound), but much larger cumulative attenuation is possible for a duct mode making many interactions with perforated walls.

Much further discussion of this mechanism is given by Howe (1980*a*) and Bechert (1980). Howe gives a general result for the rate Π_T at which energy is lost by an acoustic wave (with particle velocity $\mathbf{u}$) to a vortical field with vorticity $\boldsymbol{\omega}$, $\boldsymbol{\omega} = \operatorname{curl} \mathbf{v}$, namely

$$\Pi_T = \int \rho_0 (\boldsymbol{\omega} \wedge \mathbf{v}) . \mathbf{u} \, d^3\mathbf{x} \tag{3.5}$$

(this valid for low Mach numbers) and suggests that trailing-edge interactions always lead to an attenuation of the incident acoustic wave. Rienstra (1981) shows that this is not always true at finite Mach numbers, even for the simple case of plane waves impinging on a flat plate trailing edge with the same subsonic flow on both sides; for certain ranges of Mach number and propagation direction the power flux out of the downstream wake is positive, implying that the interactions of the acoustic and vortical disturbances with the edge generate more sound than is absorbed by the Bechert–Howe mechanism.

4. The Kutta condition in unsteady trailing-edge flows

Continuous solutions for subsonic inviscid flow around a body with sharp edges are invariably singular at those edges, the velocity there becoming infinite as some inverse fractional power of distance, $|\mathbf{x}|$, the power depending on the wedge angle at the apex. For flat plate problems the least singular solution usually has a $|\mathbf{x}|^{-\frac{1}{2}}$ singularity. If there is no mean flow, the pressure jump Δp across the plate will then vanish like $x^{\frac{1}{2}}$, but will become infinite like $x^{-\frac{1}{2}}$ in the presence of mean flow.

If the solutions are to be finite at *any* of the edges present in the flow field, then those solutions must necessarily be discontinuous across some surface extending from the edge to infinity – or must violate the radiation condition at infinity. Such a surface of discontinuity must be a vortex sheet, across which the tangential velocities jump, but across which the particle displacements and pressure are continuous. If there is no mean flow, as there frequently is not in classical acoustic diffraction problems, then there is no preferred direction along which a vortex sheet could emanate from an edge, and no mechanism in linear theory for the transport of vorticity away from the solid surface and onto the vortex sheet. One then just has to accept the continuous, singular diffraction fields, and hope that the least singular of them will actually be consistent with an acceptable structure on some 'inner' length scale provided either by linear viscous effects or, in inviscid flow, by smooth fine-scale geometry of the edge. This has actually been shown to be the case, by Alblas (1957) and Crighton & Leppington (1973), respectively. Although not couched in this language, Alblas' results can be interpreted as showing that the least singular continuous solution can be regarded as an 'outer' solution which can be matched to an inner incompressible flow governed by the linear time-dependent Stokes equations, the inner region being a Stokes layer of the usual thickness, $(\nu/\omega)^{\frac{1}{2}}$, all round the body. The velocity components in the Stokes layer *vanish* at the edge, and become large like $|\mathbf{x}|^{-\frac{1}{2}}$ as one recedes from the edge into the outer region. In the case of inviscid flow with smooth fine-scale geometry the situation is even simpler; the inner region scales on a length characteristic of that geometry, and in it the flow is nonlinear but governed by analytic solutions of Laplace's equation, these again matching the singular velocities on the outer scale.

In the presence of mean flow the inner edge problem has far more structure, and can only be understood with the aid of Stewartson's (1969) triple-deck theory. Consider, for definiteness, the flow past the trailing edge of a splitter plate extending to infinity upstream. With the same mean flow on both sides of the plate, the vortex sheet occupies the downstream extension of the plate, and the perturbation vorticity is concentrated on this surface and convected as a frozen pattern with the mean flow speed. The time-dependent potential flow equations, together with pressure and displacement continuity requirements across the vortex sheet, have, at each frequency ω, an eigensolution $\exp(-i\omega t)\,\phi_E(\mathbf{x})$ satisfying a radiation condition everywhere at infinity and with $|\mathbf{x}|^{-\frac{1}{2}}$ velocity and pressure singularities at the edge. Given, on the other hand, some external excitation in the form of an acoustic source, an incident gust, or a plate oscillation, for example, a *continuous* solution to the forced problem can be found, also with $|\mathbf{x}|^{-\frac{1}{2}}$ singularities, and radiating at infinity. A 'Kutta condition' is said to be satisfied when that multiple of the eigensolution is added to the continuous solution which removes the singularities from velocity and pressure, and this then fixes the strength of the vorticity wave convected into the 'wake'. In the absence of a Kutta condition, the general solution is not uniquely determined in terms of the forcing, and is both singular at the edge and discontinuous across the wake, so that it is natural to argue that physical effects so far neglected will in fact single out either the continuous or the non-singular solution. These two solutions differ radically in their surface loading and in their acoustic far field, and the matter of which is selected, as a function of Reynolds, Mach and Strouhal numbers and of dimensionless forcing amplitude, mean flow incidence, etc., is therefore of crucial importance in unsteady aerodynamics and aeroacoustics.

The case of non-zero, but different, mean flows on the two sides of the plate is similar, except that now the continuation of the plate is a *mean* vortex sheet, and perturbations to it now amplify in space as Helmholtz instability waves. At each frequency, the general solution will have a vortex sheet displacement $\exp(-i\omega t)\,\eta(x)$ behaving like $x^{\frac{1}{2}}$ near the edge, with $|\mathbf{x}|^{-\frac{1}{2}}$ velocity and pressure singularities in both streams, and for a particular choice of eigenfunction one can obtain $\eta(x) \sim x^{\frac{3}{2}}$ with velocities and pressures finite at the edge.

When one of the mean flows is zero, as for a jet exhausting into static fluid, a third type of edge behaviour is possible, as argued by Orszag & Crow (1970). They discussed only the eigenfunction, which itself can only be made to satisfy a Kutta condition if the pressure in the moving fluid increases like $|\mathbf{x}|^{\frac{1}{2}}$ toward upstream infinity. When considering the no-Kutta-condition solution, they argued that it would not be realistic to allow the vortex sheet to bend down into the static fluid, for in a real fluid this would immediately produce a flow separation from the plate upstream of the edge – whereas the situation in mind is one in which any separation is confined to the immediate vicinity of the edge. The downward flapping can be avoided by adding a steady flow with parabolic vortex sheet displacement of appropriate magnitude (such a displaced sheet sustaining no pressure jump in inviscid theory). This solution, involving a rectified, rather than full, Kutta condition, still has singularities at the edge in the moving fluid, but one can argue that these singularities should actually lie inside the steady parabola, and that they have been shifted into the flow domain by the linearization.

Experimental work on the Kutta condition is generally both confused and confusing. It is often claimed that great delicacy is called for in such experiments, in order to resolve the flow structure very close to the edge. Although such studies are of great intrinsic interest, they seem to me frequently to imply a complete misconception as to the nature of the Kutta condition and what measurements are needed in order to validate it. Before one can even talk about a Kutta condition one must have in mind a situation in which both mean and unsteady flows remain unseparated up to some small viscous length from the edge, so that an outer modelling of the flow by potential regions and a thin downstream wake is appropriate. The Kutta condition then asserts that the potential flow solution will behave in a certain manner as the edge is approached on length scales small compared with the outer scale, but still large compared with any relevant viscous lengths. This point, though perhaps obvious, has been misunderstood so many times that it is worth repeating; a knowledge of the inner viscous structure at the trailing edge does not necessarily clarify the Kutta condition issue, and spatial resolution very far below the inviscid length scale is not required in order to arrive at a condition applicable to the outer flow.

This matter is well exemplified by the experiments of Bechert & Pfizenmaier (1973, 1975*a*), which are of particular interest in aeroacoustics. The authors used a round jet emerging from a nozzle into static fluid to produce the mean shear layer, and perturbed the flow in a controlled manner by a loudspeaker in the plenum chamber. This forced situation is one for which a Kutta condition can be imposed without any unphysical behaviour far upstream, and the intention was to determine the form of the dividing streamline (i.e. the 'vortex sheet') close to the nozzle exit. In the first series of experiments (1973), velocities were measured in the jet stream by hot-wire anemometry, and pressures in the static fluid by a small microphone, the use of a phase reference allowing the velocity and vortex sheet displacement fields to be found as functions of space at a series of points in the phase. In terms of the boundary-layer momentum thickness θ_e at exit, hot-wire measurements were made of the streamwise velocity as a function of the transverse co-ordinate y for downstream distances x as small as $\frac{1}{2}\theta_e$, while microphone measurements in the static fluid used values of x down to about $3\theta_e$. For values of x greater than these, but small on the hydrodynamic length scale, a clear picture was seen.

First, the steady part of the dividing streamline did not have a non-zero value, as it would if the rectified Kutta condition were satisfied. Second, the hot-wire measurements showed no peak in the unsteady velocity in the neighbourhood of the edge. It was also shown that the transverse pressure gradient $\partial p/\partial y$ vanished close to the edge; while this third point is consistent with a full Kutta condition, it is also consistent with a no-Kutta-condition solution, for in the still fluid the latter has pressure varying like $|\mathbf{x}|^{\frac{3}{2}}$. It is not possible to argue that the $|\mathbf{x}|^{\frac{5}{2}}$ variation for the full Kutta condition pressure fits the data better than $|\mathbf{x}|^{\frac{3}{2}}$, but the overall position is certainly that, for the particular parameter values chosen, the potential flows satisfied the full Kutta condition in these experiments.

Dissatisfied with the resolution obtainable with the microphone in particular, Bechert & Pfizenmaier (1975*a*) then went on to devise a much more sophisticated optical compensation technique for the direct measurement of the shape of the dividing streamline. What this technique involves is irrelevant here; it did, however, enable Bechert & Pfizenmaier to measure the streamline deflection on almost the smallest viscous scales downstream of the nozzle edge at Reynolds numbers (based on diameter D) $UD/\nu = 10^4$, 5×10^4 and 10^5 and over a range of Strouhal numbers (fD/U) from 0·4 to 2·2. A typical result (their figure 11) is reproduced here as figure 2. This figure, and all the others, shows the presence, very close to the edge, of a parabola-shaped portion of the deflection curve, to which attention was drawn by Bechert & Pfizenmaier (italics on p. 142). This parabolic behaviour takes place, however, deep inside the viscous layers, must not be confused with the parabolic behaviour possible in the potential flow, and must not be interpreted – as it often has been – as implying that the potential flow does not satisfy a Kutta condition.

A (to me) very convincing explanation of the Bechert & Pfizenmaier measurements throughout the viscous regions, and a substantial clarification of the Kutta condition, has recently been given by Daniels (1978). His work was the fourth contribution in a series of natural developments, beginning with Stewartson's (1969) theory of triple-deck structure at a trailing edge in steady flow without incidence, proceeding (Brown & Stewartson 1970) to the case of steady flow past an airfoil at incidence, thence to the airfoil in oscillating or plunging motion (Brown & Daniels 1975) and finally to

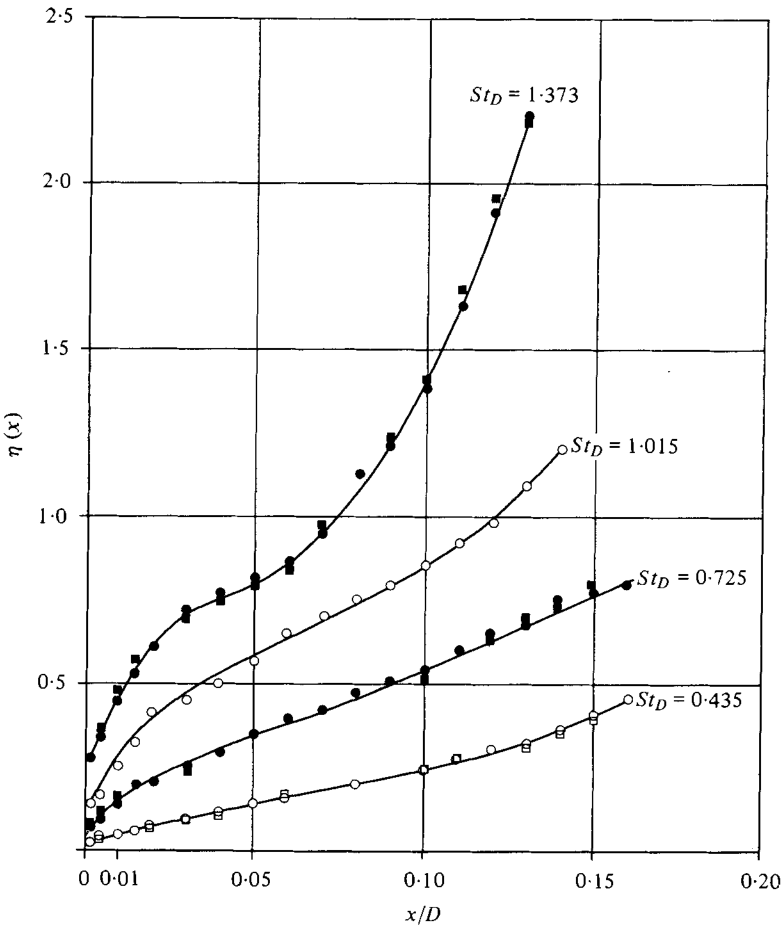

FIGURE 2. Optical compensation measurements of jet deflection envelope $\eta(x)$. $Re_D = 10^5$, various St_D. ○, ●, correspond to forcing level of 118 dB; □, ■, 108 dB. (From Bechert & Pfizenmaier (1975*a*).)

the case of unsteady perturbations to different mean flows on either side of a splitter plate. Daniels takes the outer flow to be given by the full Kutta condition eigensolution of Orszag & Crow (1970), disregarding the fact that the pressure grows algebraically upstream on the grounds that only the local edge behaviour is important (and would be similar if a forced problem were considered). Although the splitter plate is semi-infinite, it is assumed (realistically, from the point of view of experiments) that only a finite length l upstream from the edge is subject to the no-slip condition, thus giving a well-defined Blasius boundary layer just upstream of the edge in the moving fluid. This then defines a Reynolds number Ul/ν, written as ϵ^{-8}, with $\epsilon \ll 1$, in terms of which Daniels assumes that the dimensionless fluctuation amplitude in the outer flow is $O(\epsilon^{\frac{7}{4}})$ and that its reduced frequency $\omega l/U$ is $O(\epsilon^{-2})$. When the parameters are ordered in this way, a consistent, though extremely complicated, viscous flow is deduced, illustrated in Daniels' own figure 1 which is reproduced here as figure 3. The consistency of the scheme depicted depends on the construction (or at least existence) and matching of solutions in each of the many regions. Daniels achieved this for most of them, with the exception of region 11 (lower part of the triple deck), where the

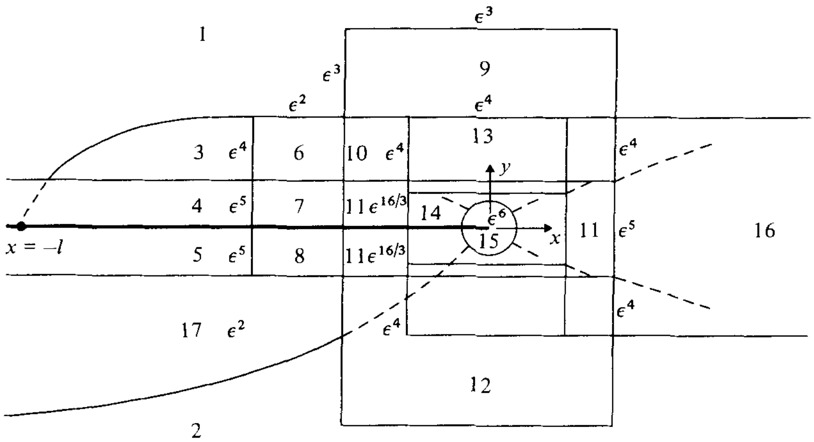

FIGURE 3. Multi-deck structure of the viscous flow at a trailing edge. Outer inviscid flow: 1, $y > 0$; 2, $y < 0$. 3, Blasius boundary layer. 4, 5, Stokes layers. Fore-deck: 6, main; 7, lower, $y > 0$; 8, lower, $y < 0$. Triple deck: 9, upper, $y > 0$; 10, main; 11, lower; 12, upper, $y < 0$. Inner region: 13, main; 14, sublayer. 15, Full Navier–Stokes region. 16, Mixing layer. 17, Displacement boundary layer. (From Daniels (1978).)

unsteady nonlinear boundary-layer equations have to be solved (which Daniels does solve in closed form in a small-amplitude limit, linearizing about a mean flow with uniform shear) and the very small region 15 where the full (but time-independent) Navier–Stokes equations apply with complicated matching conditions at infinity.

The inner layer of the triple deck has horizontal scale ϵ^3 and lateral scale ϵ^5, and, in terms of variables (x_2, y_2) which are $O(1)$ in this region, Daniels shows that the dividing streamline from the edge has equation

$$y_2 = 0{\cdot}895(x_2/a(t))^{\frac{1}{3}}, \tag{4.1}$$

where $a(t) > 0$ and, for small perturbation amplitudes,

$$a(t) = 1 + \alpha \cos \omega t, \tag{4.2}$$

with $\alpha \ll 1$. This result does not, of course, apply in region 15, where we expect the full Navier–Stokes equations to eliminate the infinite slope of (4.1) at the edge. Equation (4.1) does indeed show that in the neighbourhood of the edge the dividing streamline never turns down into the static fluid in $y < 0$ but, as Daniels remarks, that is an inherent property of the viscous and nonlinear equations and is not a condition to be imposed on the potential flow, although it does confirm the intuitive idea which Orszag & Crow evidently felt should be embodied in the solution. The result shows, secondly, that for small amplitudes the dividing streamline oscillates symmetrically on either side of the steady displacement

$$y_2 = 0{\cdot}895 x_2^{\frac{1}{3}}, \tag{4.3}$$

which it is suggested corresponds to the 'parabolic' behaviour observed on the smallest scales by Bechert & Pfizenmaier (1975*a*). Naturally enough, as $x_2 \to \infty$ and we come out of the triple deck, the dividing streamline has precisely the $y \sim x^{\frac{3}{2}}$ variation required to match the inner limit of the full Kutta condition solution. This behaviour can be observed in figure 2 above, on scales much larger than those for the 'parabolic' or $x^{\frac{1}{2}}$ variation.

Daniels also considers the viscous structure required by taking the no-Kutta-condition solution, and shows that, for amplitudes $O(\epsilon^{\frac{7}{2}})$ as before, no viscous structure can be found, implying that at these amplitudes the flow must separate from the plate well upstream of the edge. If the amplitudes are much smaller, $O(\epsilon^{\frac{13}{2}})$, a consistent viscous structure can be found if a solution exists in the Navier–Stokes region 15, which is of extent $O(\epsilon^6)$. That existence question has not been settled, but it seems reasonable that in the absence of forcing a non-separated oscillatory flow could exist at sufficiently small amplitudes, and for that the only possible representation of the outer flow would be the no-Kutta-condition solution.

What if the amplitudes and frequencies are smaller or larger than those assumed by Daniels? A complete answer has not been given, but one can perhaps anticipate that at lower amplitudes and/or frequencies the picture will remain essentially unchanged (eventually degenerating into perturbations of a steady flow, or into a quasi-steady flow). On the other hand, the multi-deck structure represents a delicate balance in which the adverse pressure gradients set up by external mechanisms (incidence, or free-stream perturbations) and tending to produce flow separation can be offset by a favourable pressure gradient set up locally by the sudden change in boundary conditions at the trailing edge. This balance cannot be maintained at substantially larger amplitudes or frequencies, and it is likely then that flow separation will take place and the assumed potential flow modelling will be inadequate.

In the case of equal mean flows a full Kutta condition can be imposed only when there is external forcing, and then with certain distinguished scalings on amplitude and frequency it can be shown that the solution under that condition can be matched to an acceptable inner viscous flow. (See Brown & Daniels (1975) for excitation by airfoil oscillation, Rienstra (1981) for that due to an acoustic source. Rienstra also shows that the sound field diffracted by the airfoil has directivity properties strongly dependent on the trailing-edge condition, and claims that these features are to be seen in the diffracted field visualizations reported by Heavens (1978).) For the no-Kutta-condition solution, Daniels casts some doubt on the existence of an acceptable viscous substructure, a point which needs to be resolved as there are experiments (Davis 1975) in which the acoustic directivity pattern generated by the vortex wake shed from a fixed airfoil with a thin, but blunt trailing edge clearly exhibited the strong preferential upstream radiation associated with the no-Kutta-condition solution. As a corollary to this remark, we might suggest that acoustic measurements in the far field may often be more easily made than flow measurements close to a trailing edge, and may, in conjunction with theory, indeed be a more decisive way of establishing trailing-edge conditions. The impressive agreement depicted in figure 1 and discussed in § 3 above, together with the experiments of Bechert & Pfizenmaier and the theory of Daniels, must be seen as strong support for the application of a full Kutta condition in most high-Reynolds-number, moderate-frequency, low-amplitude aeroacoustic problems.

5. The Kutta condition in unsteady leading-edge flows

Although the justification for applying a Kutta condition to unsteady trailing-edge flows (in some parameter ranges) is recent, the idea itself is not, having first been used more than fifty years ago. The idea of applying such a condition at the *leading* edge of a splitter plate or airfoil is, on the other hand, altogether novel; its consequences have yet to be appreciated in anything but the simplest cases – in which several puzzling features remain – and the deduction of a matching inner viscous structure appears to be a long way off.

Application of a leading-edge Kutta condition appears to have first been made, simultaneously and independently, by Goldstein (1981) and Howe (1981), with rather different views in mind. The basic problem envisaged by both authors involves the interaction of an acoustic wave with the leading edge of a semi-infinite plate in an aligned mean flow. Corresponding to the wake, supporting vortex waves or Helmholtz instabilities in the trailing-edge case, there are now viscous boundary layers (assumed thin and attached) on the plate surfaces. These are taken by Howe to be capable of supporting weakly amplifying Tollmien–Schlichting instabilities whose amplitudes are to be determined in terms of the forcing field by the elimination of the $|\mathbf{x}|^{-\frac{1}{2}}$ pressure and velocity singularities at the leading edge. Goldstein does not accept so simple a view, and rejects the boundary layer as such from having the ability to eliminate leading-edge singularities. He argues that near the edge the boundary layers are usually very thin and highly stable – in which case some condition other than one at the edge is needed to provide a 'receptivity criterion' determining the amplitudes of the Tollmien–Schlichting waves which can be observed further downstream, in terms of the forcing. (The doubly infinite shear-layer 'causality criterion' used in this context by Tam (1971, 1979) may possibly be what is needed – but that does not concern us here, beyond the fact that this criterion and Howe's edge condition each evidently relate the instability wave amplitude *linearly* to that of the forcing.) Goldstein's idea instead is that the plate must be embedded in a transversely sheared incident flow which, as it divides past the plate, remains an inherently unstable flow on scales unrelated to those of the viscous boundary layers. The best illustration of such a flow is afforded by the edge-tone configuration (see, for example, Karamcheti *et al.* 1969) in which a two-dimensional jet issuing from a high-aspect-ratio rectangular nozzle develops an instability downstream and then interacts with a symmetrically placed wedge or splitter plate. Upstream of the leading edge (of the wedge) the jet sustains a sinuous (i.e. antisymmetric, flapping) instability; downstream the divided jet remains unstable with (on each half separately) a symmetric varicose, breathing mode of instability. There is a phase difference of π between the motions on the upper and lower surfaces, as shown in figure 4 which is taken from Goldstein's paper (figure 11, plate 1).

Goldstein starts by finding a solution, bounded everywhere, for the interaction problem with an inviscid transversely sheared mean flow. The flow has different instability characteristics upstream and downstream of the edge (as noted above), but in this solution no amplifying instability waves are excited and there is singular behaviour at the (leading) edge. Call this solution $\phi_s(\mathbf{x})$ (with $\exp(-i\omega t)$ understood, and despite the fact that a potential may not always exist); this solution is not causal, i.e. is not the long-time limit of any initial-value problem, because it is not an analytic

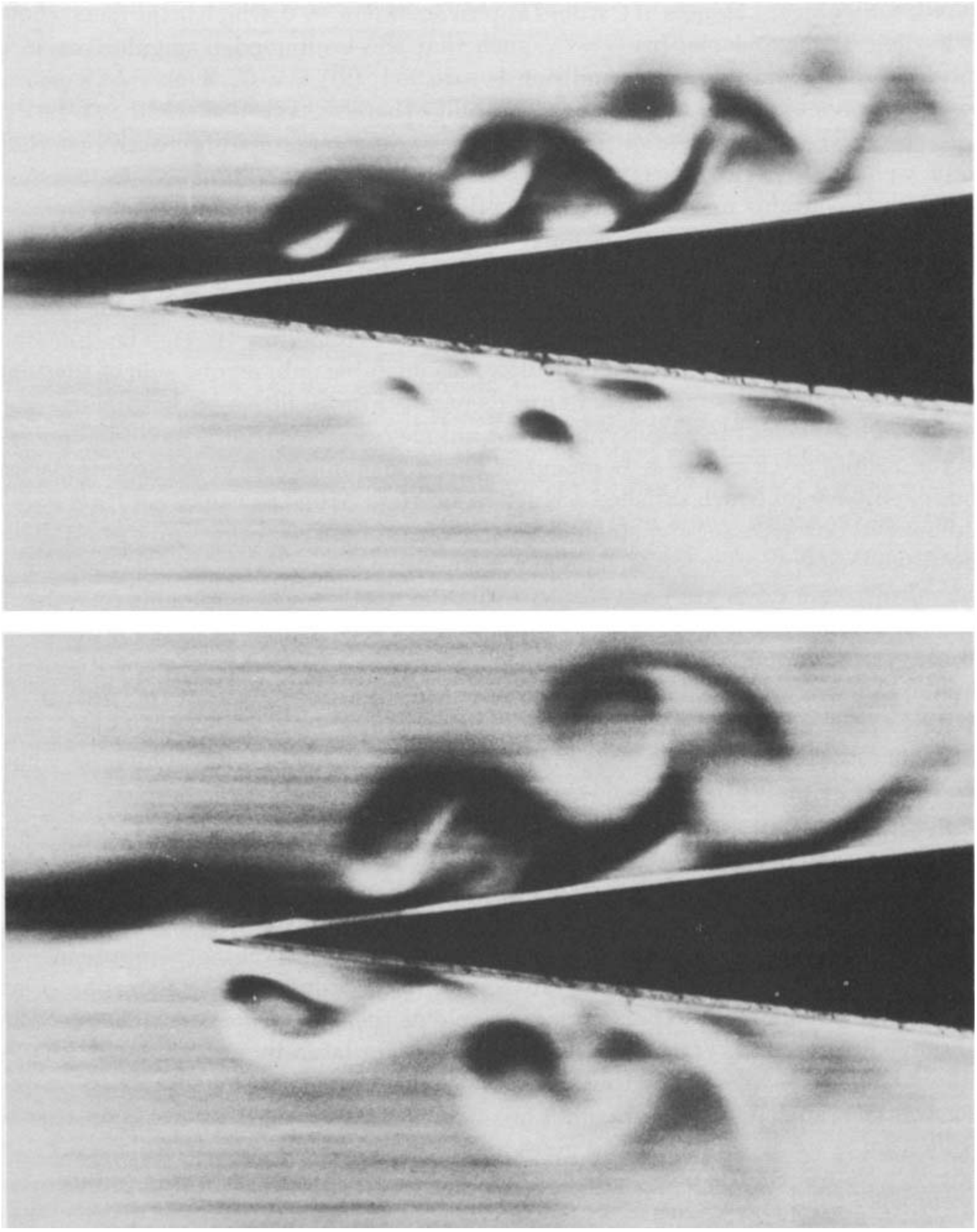

FIGURE 4. Vortex shedding downstream of a leading edge placed in an unstable rectangular jet. (Reproduced by courtesy of Dr I. Greber.)

function of ω in the half-plane Im $(\omega) > 0$. Next, an eigensolution $\phi_E(\mathbf{x})$ can be determined, with forcing absent, this being singular at the edge, non-causal, and unbounded far down the plate where it is dominated by freely propagating and amplifying instabilities. The general solution, $\phi_s(\mathbf{x}) + C\phi_E(\mathbf{x})$, is, generally, unbounded downstream, non-causal and singular at the edge and, in Goldstein's view, contains an 'incident' instability wave (for $x < 0$ say, with the half-plane in $x > 0, y = 0$) propagating toward the edge. Significant choices of C would appear to be (i) $C = 0$, which is the usual choice in leading-edge problems; (ii) $C = C_1$, such that the leading-edge singularities in ϕ_s and $C\phi_E$ cancel and a Kutta condition is satisfied; (iii) $C = C_2$, such that a pole in $\phi_s + C\phi_E$ is removed and the incident instability thereby cancelled.

In cases (ii) and (iii) instabilities are produced downstream and in each case there is the π phase difference across the plate seen in figure 4. Case (i) is non-causal because, although it contains no instability waves downstream, it does contain one incident from upstream. Case (ii) has instability waves both upstream and downstream, but no singularities for finite $\mathbf{x}$; note, however, that, in this leading-edge problem, the imposition of a Kutta condition does *not* guarantee causality (as it did, for uniform flows, in the trailing-edge problem of Crighton & Leppington 1974). The difference arises clearly because in the latter the flow was unstable only on one side of the edge, whereas in Goldstein's model there are (generally) instabilities on both sides. Case (iii), with no incident instability, gives the unique causal solution – again in contrast to the results of Crighton & Leppington where an infinity of causal solutions was found, only one of which satisfied a Kutta condition. Also, the causal solution here is unbounded downstream and singular at the edge. These remarks apply generally at low frequencies; as ω increases, however, a condition $\omega = \omega_1$ is reached beyond which the downstream flows are both stable, while the upstream flow remains unstable to sinuous modes up to a higher frequency ω_2, beyond which all flows in the problem are stable (for continuous profiles, at least). For $\omega > \omega_2$ one then has $\phi_E \equiv 0$ and the unique solution is ϕ_s with no instability wave upstream or downstream, with singularities at the edge, and with causal behaviour. For $\omega_1 < \omega < \omega_2$ (and indeed for a more complicated split of the frequency range if the mean flow is not symmetric about the line of the plate) there are no downstream instabilities, but usually one incident from upstream, and usually with singularities at the edge, etc.

How the constant C should be chosen to correspond to any particular observation is not known. Goldstein does not exclude the full Kutta condition solution (case (ii)), but seems to prefer the causal case (iii) with no incident instability. Here it seems to me that there are great difficulties of interpretation. A Fourier integral representation is obtained, and for $x < 0$ the real-axis integration path is deformed into an appropriate half-plane, leading to an expression of the field as the sum of pole contributions plus a branch line integral. The latter is *always* present for laterally unbounded flows, and cannot usefully be avoided by confining the flow between distant walls. Exact evaluations of the branch line integral cannot be achieved for any case of interest; asymptotic evaluation is straightforward and gives a field decreasing like $|\mathbf{x}|^{-\beta}$ for some $\beta > 0$, as $x \to -\infty$. Among the pole contributions may be the instability wave, proportional to $\exp\{i(k_1 - ik_2)x\}$ say, with $k_1, k_2 > 0$, but when $x < 0$ how is this to be distinguished as corresponding to a physical structure, when added to it is a branch line integral which cannot be exactly evaluated and which certainly decreases far less rapidly as $x \to -\infty$? Moreover, since the branch cut may be freely deformed in the relevant half-plane, it may be deformed across the pole (corresponding to $k_1 - ik_2$) with the result that this is no longer a pole as regards $x < 0$ (this argument does not, of course, work simultaneously for $x > 0$!). This is not to say that there are not circumstances in which there clearly *is* an instability wave incident on the leading edge; indeed all edge-tone experiments involve precisely this. However, there are difficulties in iden-

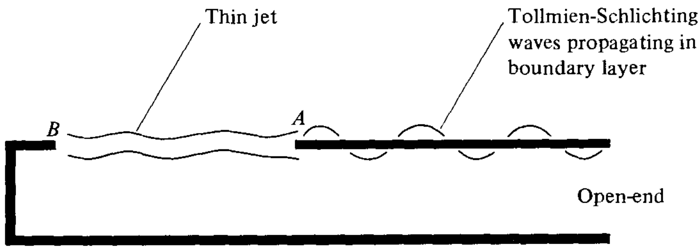

FIGURE 5. Schematic diagram of flue organ-pipe (after Howe 1981).

tifying that wave, as seen from the leading edge, which lead me to feel that at this stage it is very much an open question as to whether a Kutta condition (case (ii)) or causality (case (iii)) should be imposed.

All the solutions ((i), (ii) and (iii)) lead to the same prediction for the far-field sound at low frequencies, and this cannot therefore be used as a test of the edge condition. There might, however, be a detectable difference in the far field for the edge tone, but here there are in all theories a number of empirical inputs whose effect could mask any associated with edge conditions. That is not so in the case of instruments such as the flue organ-pipe, to which Howe (1981) applies his leading-edge condition. Howe's model of the half-plane scattering problem is much simpler than Goldstein's; Tollmien–Schlichting waves propagate in the boundary layers downstream, and generate displacement thickness fluctuations which themselves provide a boundary condition

$$\frac{\partial}{\partial y}(\phi_I+\phi') = V_{\pm}\exp(ikx)$$

applied *on* the plate, $y = 0_{\pm}, x > 0$, to the sum of the incident and scattered potentials, ϕ_I, ϕ' respectively. These satisfy the convected wave equation, and instabilities are absent from the outer flows; the only instabilities are those of the boundary layer, with complex $k = k_1 - ik_2, k_1, k_2 > 0$, and with the phase difference of π, so that $V_+ = V_-$. It is straightforward to determine ϕ' from Wiener–Hopf arguments of a simple kind, and to determine $V_{\pm}$ so that a Kutta condition holds at the leading edge. Howe shows that, when it does, acoustic energy is extracted from the mean flow by the leading-edge interaction, in contrast to the situation in § 4 where acoustic energy is lost from the incident wave in trailing-edge flows with a Kutta condition.

In application to the flue organ-pipe, Howe specifically excludes effects of upstream instability of the jet impinging on the downstream edge A of the mouth, and determines the displacement thickness fluctuations downstream by a Kutta condition at A (see figure 5). This condition is then used a second time to give a unique solution to an integral equation for the flux through the mouth – leaving singularities in velocity and pressure at the upstream *trailing* edge B! It is argued that elimination of these singularities requires (as in § 3) explicit analysis of instability waves on the thin jet spanning AB, an effect which, for the edge tone, would be expected to be more important than the downstream leading-edge condition. For the organ-pipe the assertion that the boundary-layer displacement thickness fluctuations, coupled to the resonator characteristics of the pipe, are the dominant mechanism certainly leads to an impressive outright prediction of the threshold blowing velocities of the first few modes. A corresponding theory of the edge tone, based on jet instability and trailing-edge Kutta conditions, is badly needed, and we hope (Crighton & Innes 1981) in due course to provide one.

6. Suppression of broadband jet turbulence and noise by tonal excitation

Experiments have been mentioned several times above in which a jet exhaust has been perturbed by coherent acoustic waves incident on the exit plane from within the tailpipe. In part these experiments have been directed toward an understanding of large-scale coherent structures in jets, the acoustic forcing serving to raise those structures in a phase-locked form above the fine-scale random background, and in part toward an understanding of the interaction of internally generated aeroengine noise with the nozzle and exhaust flow. The results of the experiments have shown the two aspects to be so intertwined as to be incapable of interpretation separately.

An axisymmetric jet is susceptible to three basic forms of instability. If the free shear layer at exit is laminar, an essentially two-dimensional spatial instability can develop near the exit, with properties (growth rates and phase speeds) which are well predicted by the theory of the spatial stability of a plane inviscid hyperbolic-tangent profile (Michalke 1965). If the exit shear layer is fully turbulent and the turbulence scales sufficiently small compared with the wavelength of the maximally amplified mode, one would expect the same type of instability to arise, with the *mean* profile determining the properties of the instability waves. That expectation is not always fulfilled (cf. Crow & Champagne 1971, p. 557), however, although there is generally very little difference between the laminar velocity profile and the mean profile of the turbulent layer. A possible reason may be that, when a shear layer is *tripped* into turbulence, it is too coarse-grained with respect to the instability wavelength to behave as an equivalent laminar flow. This is a point which needs further work, as control of the whole jet exhaust stems most conveniently from exploitation of the instability of the initial shear layer.

The second type of instability takes place over the range $D \lesssim x \lesssim 6D$, where D is the nozzle exit diameter. Here wavelike instability modes (with axisymmetry or with azimuthal variation) can develop on the mean profile; the waves are long (wavelength $\sim 2D$) compared with the turbulence scales, and the whole of the jet in which there is a potential core takes part in this 'jet-column' instability.

Beyond the end of the potential core, the bell-shaped mean profile becomes stable to axisymmetric modes, but remains unstable to modes with azimuthal variation (in particular to the $n = \pm 1$ spiral modes). Although there is, therefore, the possibility of further amplification far downstream of the spiral modes, that seems never to occur, in subsonic flows at any rate; it is prevented by the third type of instability which develops upstream of the end of the potential core and involves the development of rapid azimuthal variation on axisymmetric and spiral modes, and the loss of all azimuthal coherence and structure before the fully developed ($x \gtrsim 8D$) part of the flow is reached.

Many experiments on jet forcing have been carried out over the past decade, with frequencies both in the range corresponding to the initial shear-layer instability and in that for the jet-column instability, and over a wide range of forcing amplitudes (relative to the velocity or dynamic head of the exhaust jet). At sufficiently low forcing levels u_e/U, say $u_e/U \lesssim 0{\cdot}1\,\%$ with U the exit velocity and u_e the r.m.s. (filtered) exit plane fluctuation velocity, the jet response is linear and one might expect the amplifications as well as the growth rates of a large-scale jet column mode to be well predicted by linear theory for axisymmetric jets (Michalke 1971), perhaps with modifications to allow for spreading of the mean flow with axial distance as in Crighton & Gaster (1976). That comparison has not in fact been carried out for a high-Reynolds-number turbulent flow, though the reasonable agreement which is still obtained when the response is nonlinear (Crighton & Gaster 1976; Strange 1981) leads one to anticipate that the linear response is well understood. Certainly, phase speeds and transverse eigenfunction shapes are well predicted by linear theory, as is the emergence of a preferred mode of spatial instability, with Strouhal number St_D ($= fD/U$) around 0·5, the precise value being determined by the ratio of shear-layer thickness to diameter and by the flow variable and axial and transverse location considered. The generation of these large-scale waves takes place, as explained in §§ 3, 4, at the nozzle lip, where the satisfaction of a Kutta condition leads to a loss of acoustic energy from the incident waves to the instability. As the instability wave amplifies and decays, it radiates *negligible* acoustic power (in the linear response regime) in subsonic flow conditions (see figure 1) though in supersonic flow is seems likely that the instability wave itself is a powerful source of radiation.

At forcing levels $u_e/U > 0{\cdot}1\,\%$, and with the forcing coherent across the exit plane as in a plane acoustic wave, the instability wave response is significantly nonlinear, and very dramatic changes take place in the character of the internal turbulence fluctuations and the far-field noise. Conflicting evidence comes from papers published to date, though most of the results fall qualitatively into one of two categories described in this and the next section. In no single experiment, or set of experiments on the same rig, have both types of behaviour been observed in different parameter regimes, and it is difficult to find a condition which decisively separates published data into one category or the other. The two categories could reflect differences in the type of initial shear layer (laminar, transitional, fully turbulent), or in the level of free-stream turbulence, or in the Mach number, or the Reynolds number, or some combination of all these parameters and more. It seems to me, however, that presently available data are best sorted into one category or the other according to whether the jet Reynolds number $Re_D = UD/\nu$ is less than or greater than about 10^5. For $Re_D \lesssim 10^5$, the jet response (as reflected in hot-wire and pressure probe readings taken in the mixing layer, potential core and near field) to forcing at a single frequency takes the form of an amplification of the spectral levels at the forcing frequency and its integral harmonics and subharmonics, and a suppression of the broadband levels elsewhere. The same behaviour is observed in the acoustic far field (though this has not often been measured). For $Re_D > 10^5$ the response takes the form of a broadband increase, almost uniform over the entire spectrum and almost uniform over the far-field directivity, provided the forcing tone lies reasonably close to the spectral peak of the unforced response.

More detailed description of these startlingly different types of response will follow in a moment. To see that the $Re_D \lesssim 10^5$ criterion is reasonable, however, consider table 1 in which are summarized some of the results obtained by investigators of jet response at forcing amplitudes for which the instability wave behaviour (as determined by, e.g., hot-wire velocity measurements filtered in a narrow band around the forcing

Investigator	Re_D	Shear layer	St	Broadband response	Remarks
Bechert & Pfizenmaier (1975*b*)	6×10^5	T	St_D	+	0·4 %–1 % forcing
Chan (1974)	$2\cdot6\times10^5$	T	St_D	Not measured	Instability wave only measured
Crow & Champagne (1971)	$1\cdot1\times10^4$–$1\cdot2\times10^5$	T	St_D	—	1 %–2 % forcing
Favre-Marinet & Binder (1979)	$1\cdot0\times10^4$–4×10^4	L	St_D	+ (small)	Forcing up to 40 %
Jubelin (1980)	6×10^5–$1\cdot3\times10^6$	T	St_D	+	Similar results for hot jets and supersonic jets
			St_θ	—	Larger reduction for hot jets
Kibens (1979)	5×10^4–$1\cdot0\times10^5$	L	St_θ	—	—
		L	St_θ	—	Reduction at high frequencies only
Kibens (1980)	$1\cdot0\times10^5$–8×10^5	T	No forcing	Not measured	Development of coherent energy at St_D only, not at St_θ
Morrison & McLaughlin (1979)	$3\cdot7\times10^3$–$8\cdot7\times10^3$	L	St_D	—	Supersonic jets, $M_j = 1\cdot4,\ 2\cdot1,\ 2\cdot5$
Moore (1977*a*)	$1\cdot3\times10^5$–$1\cdot0\times10^6$	T	St_D	+	0·2 % maximum forcing
			St_θ	—	—
Zaman & Hussain (1981)	$1\cdot0\times10^4$–4×10^4	L	St_θ	—	—
		T	St_D–St_θ	No change	$1 \leqslant St_D \leqslant 3$

TABLE 1. Broadband jet response to single frequency forcing at amplitudes for which the instability wave behaviour is nonlinear.

frequency) is nonlinear. The third column notes whether the exit shear layer was laminar or turbulent; at the higher Re_D it was usually naturally turbulent, but at lower Re_D may have been tripped or may have been transitional. In the fourth column is an indication as to whether the frequency range covered the sensitive instability frequencies of the large-scale jet-column mode (St_D) or whether it covered the much higher range, typically by a factor 8, of instabilities on the initial shear layer (St_θ). A plus/minus sign in the next column indicates an increase/decrease in the broadband response, while the final column shows that there are great differences in the values of other parameters not always quoted (e.g. forcing level of 0·2 % for Moore, 40 % for Favre-Marinet & Binder) which may be at least as important as those reported in the papers.

A seminal paper in the area of coherent structure studies in jets is that of Crow & Champagne (1971) whose measurements cover the Reynolds number range $1\cdot1\times10^4$ to $1\cdot2\times10^5$ and who tripped the nozzle exit boundary layer to destroy the fine-scale wave pattern close to the nozzle arising from the initial shear-layer instability. Axi-

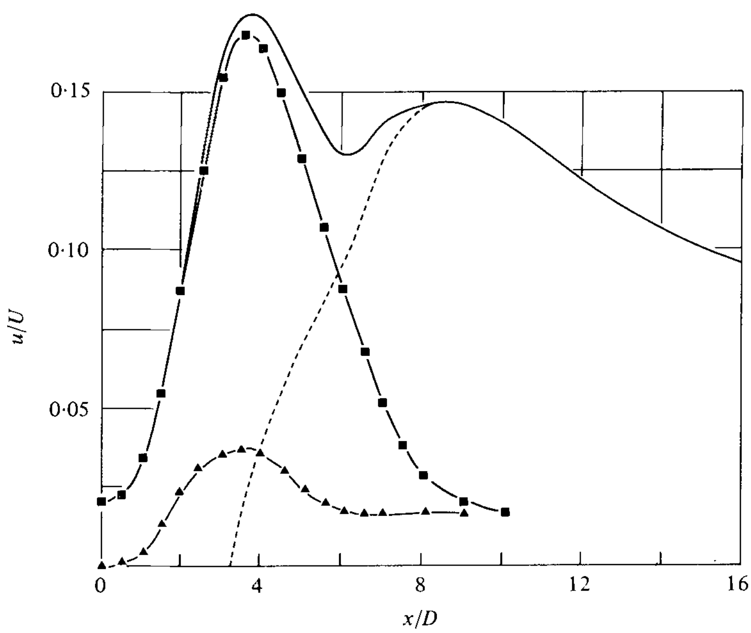

FIGURE 6. Axial profiles of r.m.s. axial velocity fluctuation on centreline. The solid curve without data points represents the overall value of u/U. Square data symbols denote the contribution of the preferred mode fundamental, $u_{0\cdot 30}/U$, triangular symbols denote the contribution of the harmonic, $u_{0\cdot 60}/U$. The dashed curve represents the residual contribution not bound up in the fundamental or harmonic. (From Crow & Champagne (1971).)

symmetric forcing from within the tailpipe was applied over a Strouhal number range around that of the preferred mode (which here had $St_D = 0\cdot 3$). The study of centreline hot-wire velocity measurements indicated that, for the typical forcing level of 1–2 % (well into the nonlinear response regime), it was possible to phase-lock the first five or six diameters of the flow, binding most of the turbulence energy into the periodic component and leaving only a small fraction of fine-scale turbulence energy not locked to the forcing. This is illustrated in figure 6 (figure 14 of Crow & Champagne's paper) which gives the variation of r.m.s. centreline axial velocity fluctuation u, normalized by the exit velocity U, as a function of downstream distance x/D, together with the velocity fluctuations $u_{0\cdot 30}$ and $u_{0\cdot 60}$ filtered in narrow bands around $St_D = 0\cdot 30$ and $0\cdot 60$ and the residual turbulence intensity not bound into these narrow-band fluctuations. As judged from these measurements (at $Re_D = 10^5$) control of the first six diameters is complete – albeit at the expense of introducing fluctuations in velocity considerably larger than in the absence of forcing (see figure 13 of Crow & Champagne 1971, where u/U at $x/D = 4$ is increased by a factor 4 by forcing at $St_D = 0\cdot 3$).

The suppression of the broadband components of u can be seen in figure 7 (figure 30 of Crow & Champagne 1971) which shows centreline axial turbulence velocity frequency spectra at $x/D = 4$ and on-axis (*a*), on the lip-line $r = \frac{1}{2}D$ (*b*), and in the near field, $r = 3D/4$ (*c*). Despite the increased fluctuation u at the axial location, it is clear that except in narrow bands around the forcing tone and its (higher) harmonics there is a reduction of the broadband spectral levels.

Similar results have been found by several investigators when the initial shear layer is laminar and the forcing is tuned to its frequency for maximal amplification. Here,

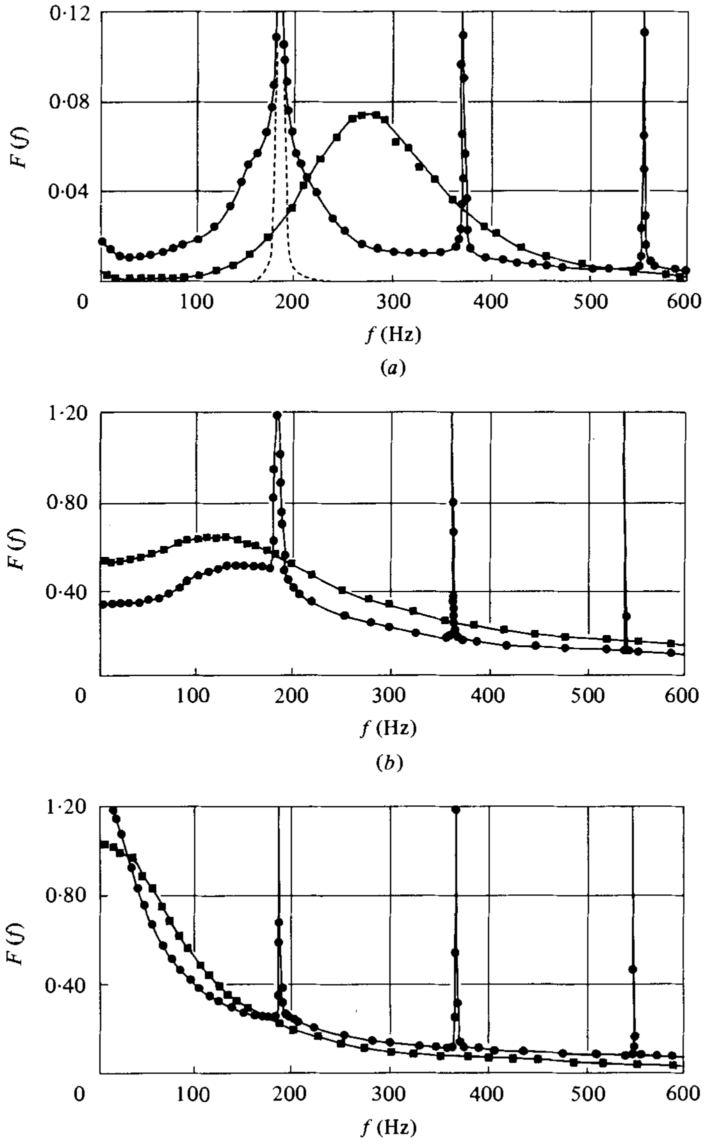

FIGURE 7. Turbulence velocity spectra at $x/D = 4$ and (a) $r/D = 0$, (b) $r/D = \frac{1}{2}$, (c) $r/D = \frac{3}{4}$. Square data points denote the unforced case and round data points the case of 2% forcing at $St_D = 0{\cdot}30$. (From Crow & Champagne (1971).)

however, a prominent feature is that tones are produced not only at the excitation frequency f_e, but at its *subharmonics* $\frac{1}{2}f_e$, $\frac{1}{4}f_e$ and even $\frac{1}{8}f_e$, and at frequencies formed by quadratic interaction of these tones (to give $\frac{1}{2}f_e + \frac{1}{4}f_e$, for example). Elsewhere the broadband levels are suppressed by the excitation. This situation is illustrated in figure 8 (figures 2 and 3 of Kibens 1979), in which the tones and suppressed broadband levels are seen both in the jet turbulence spectrum and in the far-field acoustic spectrum. There is no Doppler shift between the tone frequencies in the jet and in the far

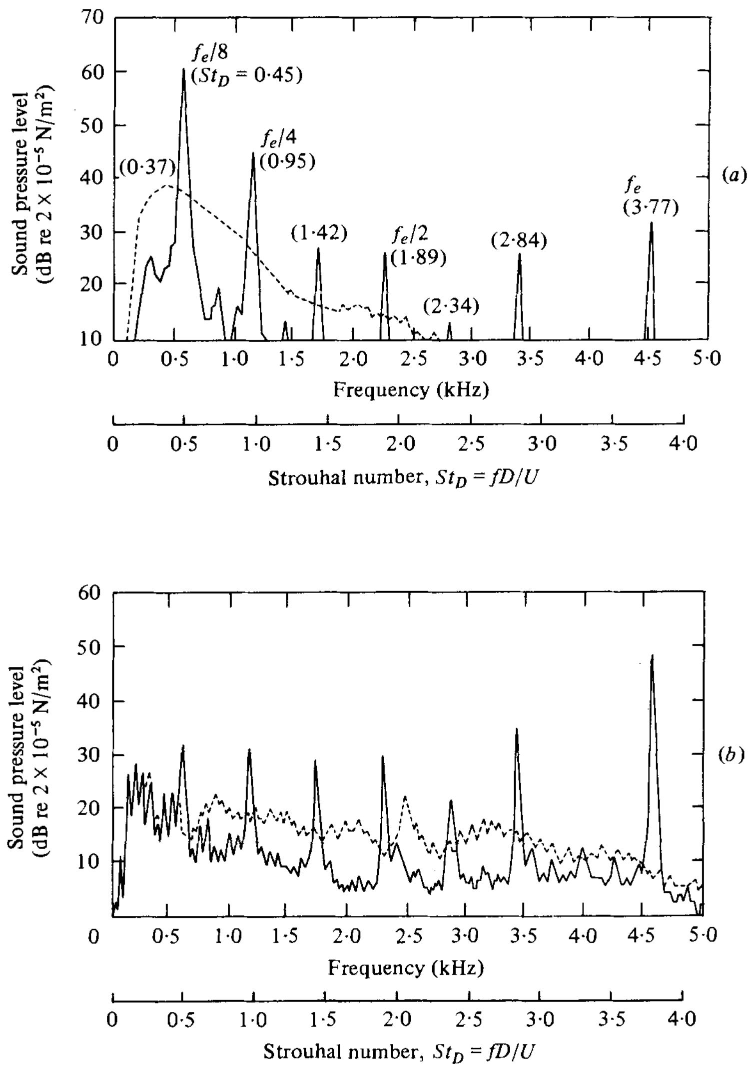

FIGURE 8. Near-field pressure spectrum (*a*) at position $(x/D, r/D) = (1, 1)$; acoustic pressure spectrum (*b*) at (50, 50). $U = 30{\cdot}5\ \mathrm{m\,s^{-1}}$, $f_e = 4525$ Hz. (From Kibens (1979).) (*a*) ——— 63·5 dB, with excitation; – – –, 55·2 dB, no excitation; (*b*) ———, 50 dB, with excitation; – – – 43 dB, no excitation.

field – which implies that the sound field at these frequencies is generated by events at locations fixed relative to the nozzle, rather than by the convected eddies usually taken to be the source of jet noise. Kibens' flow visualization indicates that these events are the succession of vortex pairings corresponding to the subharmonic frequencies. In the nonlinear response regime, the initial shear layer rolls up quickly into a succession of axisymmetric vortex rings which propagate at slightly different speeds and merge at a fixed position into larger vortices which merge, or pair, further downstream. With typical ratios (around 100) of jet diameter to initial shear-layer thickness, the vortex passage frequency $\frac{1}{8}f_e$ after three pairings is close to the preferred frequency of the jet column mode, where the length scales are too large to permit a further pairing before the end of the potential core, and this usually terminates the pairing sequence – the jet column mode thereafter amplifying and decaying, with production of integral harmonics, as in the Crow & Champagne experiments.

Naturally, the quadratic nonlinearity of the incompressible Navier–Stokes equations cannot generate a subharmonic at frequency $\frac{1}{2}f_e$ from forcing at f_e, and the process of vortex pairing apparently to 'create' energy at $\frac{1}{2}f_e$ is actually the climax to the process of amplification of the unstable subharmonic which is inevitably present at the nozzle exit, albeit generally at a very low level. This fact, that subharmonic amplification is the *cause*, rather than the *outcome*, of vortex pairing, is made very clear in recent work by Ho & Huang (1981), who forced a plane mixing layer over a range of frequencies f_e from somewhat above the frequency f_m of maximal amplification down to around $\frac{1}{5}f_m$. They showed that the response frequency f_r detected in the shear layer was equal to the forcing frequency over the range $f_m > f_e > \frac{1}{2}f_m$ (stage I), but that around $\frac{1}{2}f_m$ the response frequency jumped discontinuously to a second stage (II), starting at $f_r = f_m$ and then decreasing according to $f_r = 2f_e$. When f_e reaches $\frac{1}{3}f_m$ the frequency f_r jumps again to f_m and decreases in stage III with $f_r = 3f_e$; and so on, a total of four stages being observed. In stage N, vortices merge N at a time to produce a sparse set of large vortices as the Nth subharmonic amplifies on the shear layer; actually the merging usually first takes place in subgroups which then merge, although merging of four vortices simultaneously was observed for certain phase relationships between the subharmonic forcing at frequency f_e and the fundamental at frequency $4f_e$. Measurements of the amplification of fundamental and subharmonic by Ho & Huang make it clear that the amplification of the fundamental and its saturation under nonlinear effects do not determine where vortex merging or pairing occurs; that occurs, rather, at the locations where the *subharmonic* wave achieves its maximum amplitude.

If the initial shear layer is fully turbulent it may not be possible to excite this sequence of vortex mergings. However, subharmonic growth and vortex pairing can still take place on the jet-column further downstream. Crow & Champagne (1971) forced their jet at Strouhal number $St_D = 0{\cdot}6$, for which the preferred mode is the subharmonic $St_D = 0{\cdot}3$, and observed the vortex pairing to produce a virtual disintegration of the jet column, so great was the increase in spreading rate. Their explanation, though – that the 0·6 mode grows, pairs and then launches the preferred mode at a greatly enhanced initial level – is inconsistent with the criterion of Ho & Huang. Far from being *launched* from the vortex-pairing location, the preferred mode should have attained its peak level there.

In summary, the situation described here is very much in line with the expectations of Crow & Champagne (1971, p. 548); forcing – which amounts to control of the upstream boundary condition on the jet or shear layer – restores a degree of control to an otherwise chaotic flow, fixing the frequency, phase and azimuthal coherence of vortical wavetrains and, because of their nature as instabilities, raising them above the fine-scale background turbulence. Much of the turbulent energy in frequency bands around the forcing and its harmonics and subharmonics is locked to the forcing, and the broadband levels are reduced elsewhere by several dB. Further, in the nonlinear response regime, the unstable waves do not suffer the great amplification (by 20 or 30 dB) which they do in the linear regime – and it is therefore possible for the overall r.m.s. fluctuation to actually *decrease* with the application of forcing, though the overall decrease is always small (in terms of dB) in all the investigations quoted in table 1. A question of the greatest significance in the jet noise problem is whether the control afforded by the merging vortical instabilities, and reflected in the predominantly tonal internal and acoustic spectra, persists into the conditions of high Reynolds and Mach numbers, high temperatures and high levels of tailpipe turbulence which characterize real aeroengines. The answer, to judge from the observations of the next section, seems at the moment to be a definite 'no'.

7. Broadband amplification of jet turbulence and noise by tonal excitation

For more than a decade, jet noise prediction schemes have included significant contributions from 'excess' or 'internal' noise fields generated by unsteady processes within a jet engine (combustion, flow separation, etc.), though a quantitative connection between a specific process and the sound field has rarely been established, and excess noise has increasingly been detected at surprisingly high exhaust speeds where one would have expected jet mixing noise to dominate. The excess noise position was radically changed by the discovery, independently by Bechert & Pfizenmaier (1975*b*) and Moore (1977*a*), of a mechanism altogether more ubiquitous, subtle and insidious than internal noise *per se*.

Return to table 1 and consider the three results for $Re_D > 5\times 10^5$. Here tonal forcing at the jet-column range of instability frequencies ($0{\cdot}2 < St_D < 0{\cdot}8$) leads (in addition to large amplification of the tone component) to a large increase in the broadband turbulence spectrum and in the far field acoustic spectrum. The increase is almost uniform in frequency over many octaves, and the directivity pattern of the overall sound pressure is very similar to that of 'unexcited' jet noise – but increased in level by as much as 8 dB. If the excitation is a plane acoustic wave, then the tone protrudes from the spectrum in the far field, at least for cold jets but not necessarily for hot jets. However, the far-field tone amplitude is *linearly* related to the amplitude of the incident wave in the jet pipe, although the instability wave responds nonlinearly; and, further, the sum of the far-field tone power plus the power reflected back down the jet pipe is very close to the incident acoustic power at Helmholtz numbers above unity (roughly) and less than the incident power by as much as 15 dB at lower Helmholtz numbers – just as in the case of linear instability response. This implies that the propagation of the acoustic tone to the far-field is *linear* (at any rate at forcing levels up to about 1 %), but that interaction of the tone with the nozzle launches a nonlinear instability wave whose interaction with the background turbulence in the jet leads to the broadband amplification.

If, in the same high-Reynolds-number range, the forcing is applied at high Strouhal numbers (say $St_D > 2$) then there is no tonal response at all (as the shear layer is invariably turbulent at these Re_D), and the effect of forcing is to *reduce* the broadband turbulence and noise spectra. The reduction is small (not more than 1–2 dB for cold jets, a little more for hot jets with exit temperature $T = 600$ °K), and confined to frequencies *below* the forcing frequency.

Consider now a typical aeroengine exhaust condition with, say, $D = 1$ m, $U = 500$ m s^{-1} and $T = 900$ °K. The corresponding Re_D is about 5×10^6, and the initial shear layer is undoubtedly turbulent, as also is the flow in the jet pipe and potential core. It seems certain then that conditions will in practice favour the broadband amplification process, and indeed it has recently been shown that this process operates in many high Re_D situations (beyond the cold subsonic single-stream jets which mostly figure in table 1). In the original papers of Bechert & Pfizenmaier and Moore, broadband amplification of a plane wave pure tone excitation was demonstrated for cold subsonic jets with $Re_D > 10^5$, providing the forcing Strouhal number lies in the range $0{\cdot}2 < St_D < 0{\cdot}8$, say. Coherent velocity fluctuations of 0·1 % U are sufficient to provoke nonlinear instability wave response and broadband gain, and the amplified jet noise is, apart from the tone and its harmonics in the spectrum, "morphologically similar to ordinary jet noise, but the levels are higher" (Deneuville & Jacques 1977). As the broadband lift is a nonlinear function of the forcing velocity, and as the latter does not change linearly with U, the amplified jet noise does not have the U^8 intensity scaling for pure jet noise which is realized down to very low velocities on model rigs with very carefully controlled upstream conditions; rather, it has something like a U^6 scaling, which is often found in 'excess' aeroengine noise (excess noise being simply noise above the pure jet mixing noise level at the given engine exhaust conditions).

Flow visualization studies by Moore have shown that for cold jets there is a more or less fixed location, around $x = 3$–$4D$, at which pairing takes place between axisymmetric vortex rings, the position of pairing being localized by the excitation. The presence of the corresponding subharmonic cannot be clearly detected in Moore's spectra, nor in those of Jubelin for similar conditions; for hot jets, however, Jubelin found that a spectral peak was present at the subharmonic frequency, and that it was entirely stable and repeatable (the exit temperatures were 600 and 900 °K). Moore (1977*b*) has also used an acoustic telescope technique to show that the apparent source position for *all* frequencies is, in the case of amplified noise, around the vortex-pairing location, whereas, when the forcing is linear and the jet noise not amplified, the sources are distributed over many diameters of the exhaust, with high frequencies coming from near the nozzle, low frequencies from sources well down in the fully developed jet.

Moore (1977*a*; see figure 9 here, taken from that paper) makes brief reference to excitation of a broadband nature itself, and it has been found that broadband amplification still results in this case (although in a given experimental facility the amplification will usually be less because of weaker response of the loudspeaker system to broadband drive). Deneuville & Jacques quote several examples which show that rather uniform amplification of several dB is found on either side of the frequencies occupied by broadband forcing. They also emphasize that although in the subsonic cold jet experiments the forcing is easily discerned in the far field, where generally the tone level exceeds the overall jet noise, there are many cases in which it is impossible to detect the forcing. As an example they quote results for a jet with $M = 1{\cdot}0$, $T = 300$ °K in which the only irregularities in the spectrum are to be seen at 120° to the jet exhaust, and there they are sufficiently slight as to deserve no comment – but the jet actually suffers from broadband amplification by a pure tone. Bechert & Pfizenmaier (1976) used an array of loudspeakers feeding into the jet pipe to force a cold subsonic jet with a first-order helical mode. The frequency was such that the mode was cut off in the jet pipe and therefore not detectable in the far field, although it generated a nonlinear instability wave in the jet and broadband noise amplification very much as in the plane-wave case. The upshot of these results is that the presence

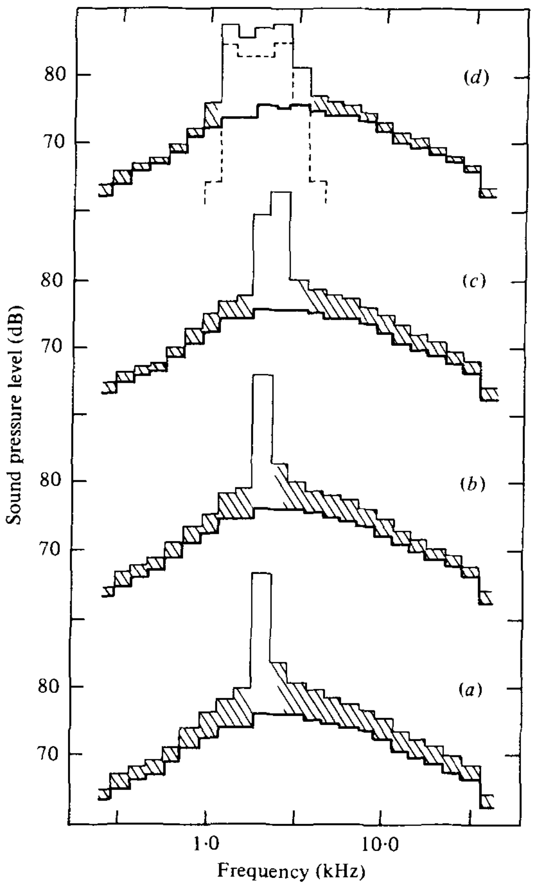

FIGURE 9. Comparison of far-field spectra (——) with and (——) without excitation, at 60° to jet axis, at jet velocity of $0{\cdot}49a_0$. (*a*) Pure tone excitation. (*b*) 30 Hz bandwidth random excitation. (*c*) 300 Hz bandwidth random excitation. (*d*) Random excitation in band from 1 kHz to 3 kHz. – – –, excitation spectrum. (From Moore 1977*a*.)

of irregularities in the far-field spectra is not a satisfactory criterion by which to judge whether jet noise amplification has taken place.

Jubelin (1980) has studied forcing effects on hot jets, with temperature up to 900 °K and Mach number 0·5. Broadband amplification results, much as for cold jets, but with the following differences: (i) the amplification is not as uniform in angle for the hot jet, being small close to the exhaust direction and much larger in the forward arc, (ii) the amplification is less uniform in frequency for the hot jet, being smaller at low frequencies than for the cold jet and generally more concentrated around the forcing frequency, (iii) the hot jet is less sensitive to excitation than the cold in the sense that the slope of the jet noise power *vs*. excitation curve is less for the hot jet, (iv) the threshold for amplification may, for the hot jet, be as much as 25 dB less in power than the jet noise power, whereas for the cold jet the excitation power must, for broadband amplification, be much closer to the jet noise power, (v) as noted before, a subharmonic at half the forcing frequency is a pronounced and stable feature of the spectra of excited hot jets, but not of cold jets, (vi) the broadband decrease which accompanies high-frequency forcing is more noticeable on hot jets than cold (though in neither case does it exceed 2–3 dB). These results are of particular importance, as model jets at these high temperatures and with tailpipe excitation are the best simulation of a jet engine that can be achieved.

In the same paper, Jubelin also showed the presence of a broadband amplification for warm (i.e. exit temperature 300 °K) supersonic jets. Jets at Mach number 1·2 were excited both in underexpanded flow through a convergent nozzle and in perfectly expanded flow through a convergent–divergent nozzle. In the first case the spectra at all angles from 60° to the exhaust round into the forward arc are dominated by high levels of broadband shock-associated noise, generated by the interaction of shear-layer turbulence with the cellular wave pattern in the exhaust jet; in the latter case there is some slight evidence of shock-associated noise but the main spectral energy is associated with supersonic mixing noise (though not Mach wave radiation from supersonically convected eddies as the value of M is not high enough for this at ambient temperatures). In both cases there is strong broadband amplification under forcing at $St_D = 0{\cdot}46$, with no evidence of the forcing in the spectra and rather large amplifications at high frequencies and at high angles to the exhaust. That shock-associated noise and mixing noise should both be amplified is not surprising, as both are generated by the broadband turbulence in the jet; no account has yet been taken, however, in prediction schemes, of the fact that each may suffer broadband amplification of 5 dB or more in spectral ranges which at full scale are extremely important in the determination of perceived noise levels (PNdB). A further point which should be observed is that in none of these studies has any saturation effect been found for the broadband amplification; the maximum amplification obtained has been limited by the available drive power, and there might well be greater amplification if forcing levels higher than 0·2–1 % could be applied. On the other hand, as remarked by Moore (1977*a*), jet engine noise levels correlate reasonably from one engine to another, and far better than they correlate with scaled-up model data, which might be taken to imply that all jet engines are in fact excited to the point of saturation.

The broadband amplification effect is thus ubiquitous in the noise and turbulence of model jets; it occurs when forcing levels exceed a low threshold, over a wide range of Strouhal numbers over the temperature range 300–900 °K, and at subsonic or supersonic speeds, provided only that $Re_D > 10^5$. Under these conditions it is eliminated, or greatly alleviated, by two mechanisms – coaxial flow, and use of a convoluted suppressor nozzle. Moore & Brierley (1979) and Jubelin (1980) have studied the effects of a secondary coannular flow on the noise of an excited primary jet, with possible application not only to turbofan engines but also to the noise of a single-stream turbojet in flight. There are several mean flow parameters here and only a very limited number of configurations has been examined so far. In each case, the behaviour of the primary alone under excitation was as expected, but Jubelin found *no significant amplification of the dual-flow jet in any case*, regardless of whether the primary stream forcing was tuned to the preferred frequency of the primary or secondary stream. In the Rolls Royce experiments the amplification decreased to a minimum (effectively zero) at a secondary to primary velocity ratio of 0·5, where high-speed films showed that there was no longer any evidence of vortex pairing on the primary jet. Thus the fairly definite conclusion must be that external flow inhibits instability wave growth and prevents the vortex pairing which seems to be the source of broadband amplification. The position may be different if the forcing is injected into the secondary flow, rather than the primary, but that situation is irrelevant if the secondary flow is the infinite ambient medium in the engine-in-flight case, and less interesting anyhow in the turbofan engine case where one would expect secondary excitation to be small.

Several authors have considered tailoring the mean flow profile in a single-stream jet so as to inhibit instability wave growth. Chan & Templin (1974) used a non-uniform gauze to produce a bell-shaped profile at exit (this according to well-known results in stability theory being stable to axisymmetric disturbances), and showed that all such disturbances decayed rapidly downstream; non-axisymmetric modes can, however, still grow on the modified profile, but were not considered. Bechert & Pfizenmaier (1975*a*) showed that a simple annular ring inside the nozzle reduced the broadband amplification, though how that was achieved is not clear, as the ring does not change plane-wave forcing nor does it significantly change the mean profile. Moore & Brierley (1979) examined the forcing of a cold jet from a 6-chuted and a 7-lobed suppressor nozzle and compared the noise fields with those from their parallel circular nozzle (at the same mass flow); only very small levels of amplification were found for the suppressor nozzles as against the 8 dB or so for the plain nozzle. The explanation given is that there is no thin shear layer on which coherent structures can grow, but probably most important is the fact that the suppressor nozzles destroy all azimuthal coherence. In any event, the lack of amplification may well be the prime mechanism behind the silencing action of these nozzles (for which many different claims have been made) and may explain the great variability of effectiveness of them in different situations.

There is increasing indirect evidence for amplification phenomena in practically important cases, whether model jets, or full-scale combustion and turbine rigs, or complete engines. Many such pieces of evidence are quoted in a delightful paper by Deneuville & Jacques (1977) who point out that the model and rig tests fully represent practical conditions. Among those discussed are the following:

(1) Frequent occurrence of noise fields with characteristics of pure jet noise, but with higher levels, while a model rig was being set up and adjusted.

(2) A 2 dB broadband noise increase from a hot jet rig when the mass flows in the fuel sprays were imperfectly balanced, leading to flame instability and low-frequency parasitic tones. In the example chosen the jet was supercritical, with mixing and shock-associated noise in different angular and spectral ranges – and both fields were amplified.

(3) In this hot jet rig, a large nozzle was fitted and the combustion turned off. For a given nozzle pressure ratio the large nozzle leads to a much greater flow velocity through the combustion chamber, and to higher, though similar, noise spectra at all angles than in the case of a small nozzle scaled to the same area. In this case there is no evidence of any tones in the spectra, but it is thought that there is an amplification effect in response to large fluctuations developed in the combustion chamber.

(4) A conical nozzle flow emerging into a cylindrical ejector shroud (with only weak secondary flow) can, in some flow regimes, excite open-tube axial resonances in the shroud. These react upon the jet, causing broadband amplification distinguishable (from spectral and angular properties) from broadband noise generated by hydrodynamic interaction of the jet with the ejector.

(5) The Concorde Olympus 593 engine has a field shape generally 3–5 dB above jet noise prediction, especially in the forward arc, and though the spectra contain a lot

of mid- and high-frequency internal noise components it is possible that much of the low-frequency excess noise is amplified jet noise.

(6) A turbojet engine was run at the *same running point* with a small and a large nozzle, with the aid of upstream throttling. With the large nozzle, jet noise levels are low and what is measured is essentially internal noise (with a flat spectrum, rather different from that of jet noise). Then the internal noise levels are corrected for nozzle area and compared with the measured noise for the small nozzle – for which it is found that the spectra are like those of jet noise, but much higher in level than according to prediction. The internal noise levels are rather accurately known from the large-nozzle measurements, and cannot explain the discrepancy which, it is concluded, is caused by jet noise amplification by the internal noise.

(7) A perforated honeycomb tailpipe liner, tuned to attenuate high frequencies, was fitted to a Larzac engine on which jet noise levels were very low and internal noise levels high; the liner attenuated only the expected high frequencies. When the same liner was fitted to the Olympus 593 at a condition where the predicted jet noise and measured noise were comparable, the liner gave significant attenuations (2–6 dB) over a very wide frequency range, and in particular a uniform 2 dB attenuation below 400 Hz. Acoustic absorption by the liner is not possible at such low frequencies and the effect is attributed to a reduction of upstream forcing levels, and hence a reduction of jet noise amplification, by the liner. An ‘egg-box’ flow straightener fitted to the Olympus 593 also leads to significant broadband reductions, particularly in the forward arc, which might be expected if the action of the device is to reduce tail-pipe fluctuations in a *hot* jet.

Finally, in this catalogue of practical examples, Moore (1977*b*) shows noise levels measured on a large ($D = 0{\cdot}3$ m) hot jet rig with internal combustion noise; the spectra are similar to predicted pure mixing-noise spectra, with some irregularities at frequencies in the 200–400 Hz range characteristic of combustion noise, and up to 10 dB higher. Moore estimated the internal excitation levels by extrapolating the energy in the combustion noise spectral bump back to the nozzle exit, and then used his (cold jet) results to predict that such excitation would produce a broadband increase of 8·5 dB – rather close to the observed difference.

Enough has now been said to convince the reader of the frequent occurrence of broadband amplification in many practical cases with naturally occurring types and levels of excitation. Most cases in which its occurrence is suspected do, however, involve static single-stream jets, and the work on dual flow jets makes it appear unlikely that broadband amplification can play such a significant role in engines in flight, or on turbofan engines.

8. Concluding remarks

I hope that this article will both persuade the general reader of the vitality of *acoustics* as a branch of fluid mechanics and provide a review of topical aspects of aeroacoustics. Paradoxically, perhaps, the main theme has been the interaction of unsteady, unstable *vortical* fields with solid boundaries and with external forcing. In some aspects (notably §§ 3, 4) understanding is now satisfyingly complete, while in others (§ 7) there are really not even the rudiments of an understanding of the fluid mechanics involved, and the chances of satisfactorily predicting the acoustics are even more remote. For three very different approaches to the subject of § 7 the reader is referred to Ffowcs Williams & Kempton (1978), Morfey (1979) and Mankbadi & Liu (1980). Contributors to this *Journal* will, no doubt, continue to develop the models discussed there as well, perhaps, as introducing some of the nonlinear wave/chaotic dynamics ideas currently fashionable in geophysical fluid dynamics.

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REFERENCES

ALBLAS, J. B. 1957 *Appl. Sci. Res.* A **6**, 237.

BECHERT, D. 1980 *J. Sound Vib.* **70**, 389.

BECHERT, D., MICHEL, U. & PFIZENMAIER, E. 1977 *A.I.A.A. Paper* 77–1278.

BECHERT, D. & PFIZENMAIER, E. 1973 On the Kutta condition at the nozzle discharge edge in a weakly unsteady nozzle flow. DLR FB 71–09. English translation as *RAE Lib. Trans.* no. 1617.

BECHERT, D. & PFIZENMAIER, E. 1975*a* *J. Fluid Mech.* **71**, 123.

BECHERT, D. & PFIZENMAIER, E. 1975*b* *J. Sound Vib.* **43**, 581.

BECHERT, D. & PFIZENMAIER, E. 1976 *A.I.A.A. Paper* 76–489.

BROADBENT, E. G. & MOORE, D. W. 1979 *Phil. Trans. R. Soc.* A **290**, 353.

BROWN, S. N. & DANIELS, P. G. 1975 *J. Fluid Mech.* **67**, 743.

BROWN, S. N. & STEWARTSON, K. 1970 *J. Fluid Mech.* **42**, 561.

CARGILL, A. 1979 In *Mechanics of Sound Generation in Flows* (ed. E.-A. Müller), p. 19. Springer.

CARGILL, A. 1981 Low frequency acoustic radiation from a jet pipe: a second-order theory. (Submitted to *J. Sound Vib.*)

CHAN, Y. Y. 1974 *Phys. Fluids* **17**, 46, 1667.

CHAN, Y. Y. & TEMPLIN, J. T. 1974 *Phys. Fluids* **17**, 2124.

CRIGHTON, D. G. 1975 *Prog. Aerospace Sci.* **16**, 31.

CRIGHTON, D. G. 1979 In *Mechanics of Sound Generation in Flows* (ed. E.-A. Müller), p. 1. Springer.

CRIGHTON, D. G. & GASTER, M. 1976 *J. Fluid Mech.* **77**, 397.

CRIGHTON, D. G. & INNES, D. 1981 *A.I.A.A. Paper* 81–0061.

CRIGHTON, D. G. & LEPPINGTON, F. G. 1973 *Proc. R. Soc.* A **335**, 313.

CRIGHTON, D. G. & LEPPINGTON, F. G. 1974 *J. Fluid Mech.* **64**, 393.

CROW, S. C. 1970 *Stud. Appl. Math.* **49**, 21.

CROW, S. C. 1972 Acoustic gain of a turbulent jet. *Bull. Am. Phys. Soc.*, Paper IE 6.

CROW, S. C. & CHAMPAGNE, F. H. 1971 *J. Fluid Mech.* **48**, 547.

DANIELS, P. G. 1978 *Quart. J. Mech. Appl. Math.* **31**, 49.

DAVIS, S. S. 1975 *A.I.A.A. J.* **13**, 375.

DENEUVILLE, P. & JACQUES, J. R. 1977 *A.I.A.A. Paper* 77–1368.

DOWLING, A. P., FFOWCS WILLIAMS, J. E. & GOLDSTEIN, M. E. 1978 *Phil. Trans. R. Soc.* A **288**, 321.

FAVRE-MARINET, M. & BINDER, G. 1979 *J. de Mécanique* **18**, 356.

FAY, R. D. 1931 *J. Acoust. Soc. Am.* **3**, 222.

FFOWCS WILLIAMS, J. E. 1969 *Ann. Rev. Fluid Mech.* **1**, 197.

FFOWCS WILLIAMS, J. E. 1977 *Ann. Rev. Fluid Mech.* **9**, 447.

FFOWCS WILLIAMS, J. E. & KEMPTON, A. J. 1978 *J. Fluid Mech.* **84**, 673.

GOLDSTEIN, M. E. 1976 *Aeroacoustics*. McGraw-Hill.

GOLDSTEIN, M. E. 1981 *J. Fluid Mech.* **104**, 217.

HEAVENS, S. N. 1978 *J. Fluid Mech.* **84**, 331.

HO, C.-M. & HUANG, L.-S. 1981 Subharmonics and vortex merging in mixing layers. Submitted to *J. Fluid Mech.*

HOWE, M. S. 1975 *J. Fluid Mech.* **71**, 625.

HOWE, M. S. 1979*a* *J. Fluid Mech.* **91**, 209.

HOWE, M. S. 1979*b* *Proc. R. Soc.* A **366**, 205.

HOWE, M. S. 1980*a* *J. Sound Vib.* **70**, 407.

HOWE, M. S. 1980*b* *J. Fluid Mech.* **97**, 641.

HOWE, M. S. 1980*c* *Proc. R. Soc.* A **370**, 523.

HOWE, M. S. 1981 *Proc. R. Soc.* A **374**, 543.

HOWE, M. S. & FFOWCS WILLIAMS, J. E. 1978 *Phil. Trans. R. Soc.* A **289**, 271.

HOWE, M. S. & LIU, J. T. C. 1977 *J. Fluid Mech.* **81**, 369.

JUBELIN, B. 1980 *A.I.A.A. Paper* 80–0961.

KARAMCHETI, K., BAUER, A. B., SHIELDS, W. L., STEGUN, G. R. & WOOLLEY, J. P. 1969 *N.A.S.A.* SP 207, 275.

KIBENS, V. 1979 In *Mechanics of Sound Generation by Flows* (ed. E.-A. Müller), p. 174. Springer.

KIBENS, V. 1980 *A.I.A.A. Paper* 80–0963.

LIGHTHILL, M. J. 1952 *Proc. R. Soc.* A **211**, 564.

LIGHTHILL, M. J. 1963 *A.I.A.A. J.* **1**, 1507.

LILLEY G. M. 1972 AFAPL TR-72-53, vol. 4.

MANKBADI, R. & LIU, J. T. C. 1980 *Phil. Trans. R. Soc.* A **298**, 541.

MICHALKE, A. 1965 *J. Fluid Mech.* **23**, 521.

MICHALKE, A. 1971 *Z. Flugwiss.* **19**, 319.

MÖHRING, W. 1978 *J. Fluid Mech.* **85**, 685.

MOORE, C. J. 1977*a* *J. Fluid Mech.* **80**, 321.

MOORE, C. J. 1977*b* In *Structure and Mechanisms of Turbulence*, vol. II (ed. H. Fiedler), p. 254. Lecture Notes in Physics, vol. 76. Springer.

MOORE, C. J. & BRIERLEY, D. H. 1979 In *Mechanics of Sound Generation in Flows* (ed. E.-A. Müller), p. 48. Springer.

MORFEY, C. L. 1979 In *Mechanics of Sound Generation in Flows* (ed. E.-A. Müller), p. 12. Springer.

MORRISON, G. L. & MCLAUGHLIN, D. K. 1979 *J. Sound Vib.* **65**, 177.

MUNT, R. M. 1977 *J. Fluid Mech.* **83**, 609.

MUNT, R. M. 1981 Acoustic transmission properties of a jet pipe with subsonic jet flow, Parts I and II. Submitted to *J. Sound Vib.*

OBERMEIER, F. 1979 *Acustica* **42**, 56, 62.

ORSZAG, S. A. & CROW, S. C. 1970 *Stud. Appl. Math.* **49**, 167.

PHILLIPS, O. M. 1960 *J. Fluid Mech.* **9**, 1.

POWELL, A. 1964 *J. Acoust. Soc. Am.* **36**, 177.

RAYLEIGH, J. W. S. 1877 *Theory of Sound*, 2 vols, Macmillan. Reprinted by Dover, 1945.

RIBNER, H. S. 1962 Aerodynamic sound from fluid dilatations. *UTIA Rep.* no. 86, AFOSR TN 3430.

RIENSTRA, S. W. 1981 Sound diffraction at a trailing edge. *J. Fluid Mech.* (in press).

STEWARTSON, K. 1969 *Mathematika* **16**, 106.

STRANGE, P. J. R. 1981 Ph.D. Thesis, University of Leeds.

TAM, C. K. W. 1971 *J. Fluid Mech.* **46**, 757.

TAM, C. K. W. 1979 *J. Fluid Mech.* **89**, 357.

TAM, C. K. W. & MORRIS, P. J. 1980 *J. Fluid Mech.* **98**, 349.

TAYLOR, G. I. 1910 *Proc. R. Soc.* A **84**, 371.

ZAMAN, K. B. M. Q. & HUSSAIN, A. K. M. F. 1981 Turbulence suppression in free shear flows by controlled excitation. *J. Fluid Mech.* **103**, 133.